Dynamics and Control of Adaptive Networks

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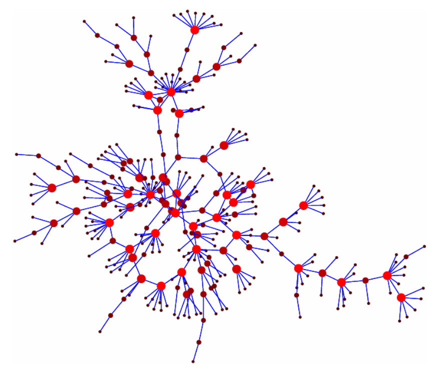
Sponsors: Office of Naval Research. Army Research Office, Air Force Office of Scientific Research, National Institutes of Health

Graph Exploitation-MIT LL 2010

Background: Networks

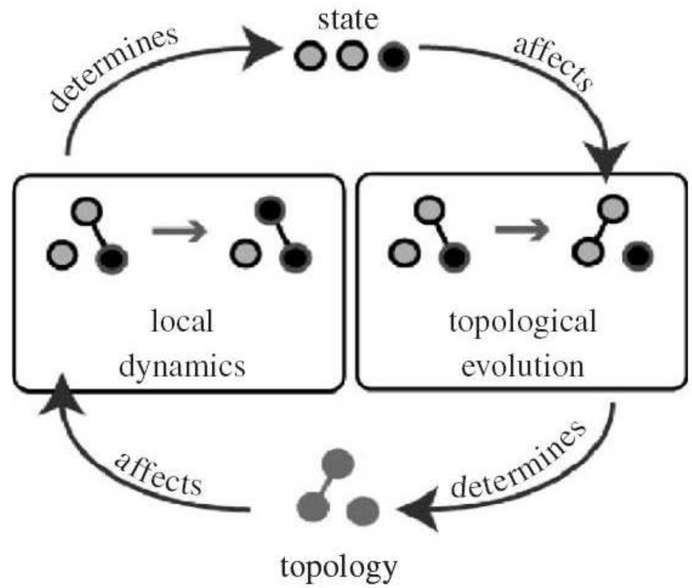
- Interconnected systems are often modeled as a network
- Structure of static networks has been well studied (Albert and Barabasi 2002, Newman 2003)
- Some aspects of network dynamics have been studied (usually link dynamics **or** node dynamics)
  - Network growth
  - Synchronization dynamics on nodes (Arenas et al. 2008)



M. E. J. Newman,
SIAM Review 45, 167-256 (2003).

Background: Adaptive Networks

- In real networks, both the nodes and links may change in time – Dynamic networks
- Often, node dynamics affects network geometry while network geometry affects node dynamics – Adaptive networks
- Feedback loop interaction
- Adaptive networks have many potential applications
  - Human social networks
  - Self-healing communications networks
  - Swarming of autonomous agents
  - Biological networks (e.g., food webs)



Gross and Blasius
J. Roy. Soc. Interface 2008

Adaptive Epidemic Social Network

- **Problem**: How do you model an epidemic spreading in a social network?
  - People may change social behavior / social connections to avoid exposure to a disease – Adaptive network
- **Approach**: Rewire the network so non-infected people avoid contact with infected people

Gross et al, PRL 96: 208701, 2006
SIS model on adaptive network

Shaw and Schwartz PRE 77: 066101, 2008
SIRS model

Shaw and Schwartz 2009,
in Adaptive Networks:
Theory, Models and Applications,
eds. Gross and Sayama
SIRS model

Schwartz and Shaw, Physics 3: 17, 2010
Adaptive networks and control

Outline

- Epidemic model on an adaptive network
  - Bifurcation structure
  - Network geometry
  - Fluctuations
- Control strategy - model with vaccination
  - Bifurcation structure
  - Lifetimes
  - Effect on degree

Rules for Network Dynamics

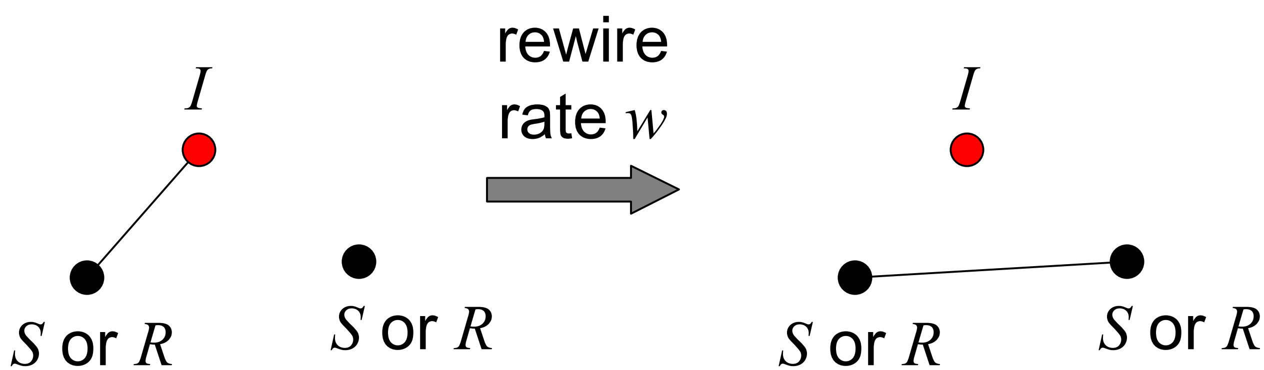
S: susceptible
I: infected
R: recovered

p: infection rate
r: recovery rate
q: resusceptibility rate
w: rewiring rate

Epidemic dynamics:

$$\boldsymbol{S} \xrightarrow{p} \boldsymbol{I} \xrightarrow{r} \boldsymbol{R} \xrightarrow{q} \boldsymbol{S}$$

Network dynamics—rewiring:



Run Monte Carlo simulation for $N=10^4$ nodes, $K=10^5$ links

Mean field approximation

$$S \xrightarrow{p} I \xrightarrow{r} R \xrightarrow{q} S$$

- Node dynamics—depends on node pairs (links)

$$\frac{dN_S}{dt} = qN_R - pN_{SI}$$

$$\frac{dN_I}{dt} = pN_{SI} - rN_I$$

- Link dynamics—(depends on triples)

$$\frac{dN_{SI}}{dt} = qN_{RI} + \boxed{2pN_{SSI}} - rN_{SI} - p(N_{SI} + \boxed{N_{ISI}}) - \boxed{wN_{SI}}$$

Mean field approximation

- Make an approximation for three-point terms to close the equations (moment closure approximation)

$$\left(P_{ABC} \approx \frac{P_{AB} P_{BC}}{P_B} \right)$$

Nodes:

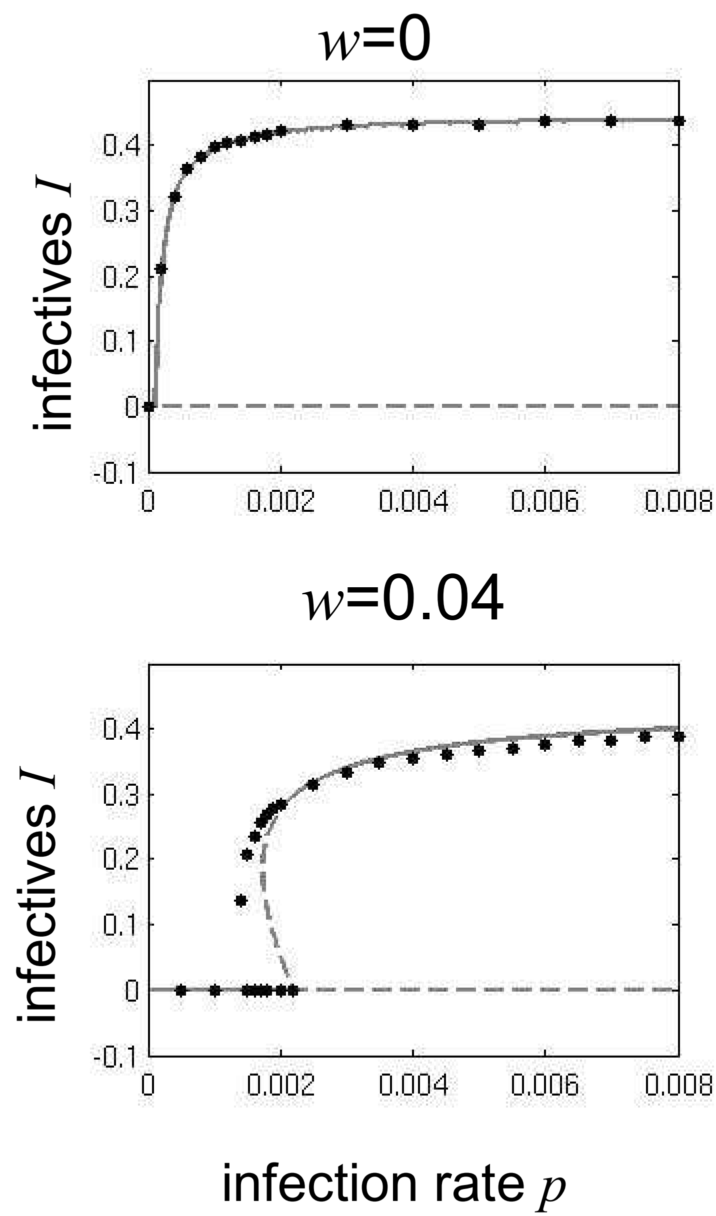
$$\dot{P}_S = qP_R - p\frac{K}{N}P_{SI}$$
$$\dot{P}_I = p\frac{K}{N}P_{SI} - rP_I$$
$$\dot{P}_R = rP_I - qP_R$$

Links:

$$\dot{P}_{SS} = qP_{RS} + w\frac{P_S}{P_S + P_R}P_{SI} - 2p\frac{K}{N}\frac{P_{SS}P_{SI}}{P_S}$$
$$\dot{P}_{SI} = 2p\frac{K}{N}\frac{P_{SS}P_{SI}}{P_S} + qP_{IR} - rP_{SI} - \boxed{wP_{SI}} - p\left(P_{SI} + \frac{K}{N}\frac{P_{SI}^2}{P_S}\right)$$
$$\dot{P}_{II} = p\left(P_{SI} + \frac{K}{N}\frac{P_{SI}^2}{P_S}\right) - 2rP_{II}$$
$$\dot{P}_{SR} = rP_{SI} + w\frac{P_R}{P_S + P_R}P_{SI} + 2qP_{RR} - qP_{SR} - p\frac{K}{N}\frac{P_{IS}P_{SR}}{P_S} + w\frac{P_S}{P_S+P_R}P_{IR}$$
$$\dot{P}_{IR} = 2rP_{II} + p\frac{K}{N}\frac{P_{IS}P_{SR}}{P_S} - qP_{IR} - rP_{IR} - \boxed{wP_{IR}}$$
$$\dot{P}_{RR} = rP_{IR} - 2qP_{RR} + w\frac{P_R}{P_S + P_R}P_{IR}$$

Bifurcation structure

- Rewiring leads to bistable behavior
- As rewiring rate w increases, larger infection rate p is needed for disease to persist



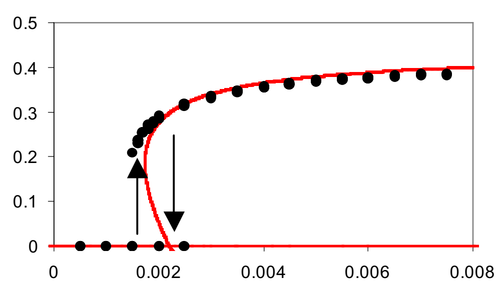
Monte Carlo •

mean field —

q=0.0016, r=0.002

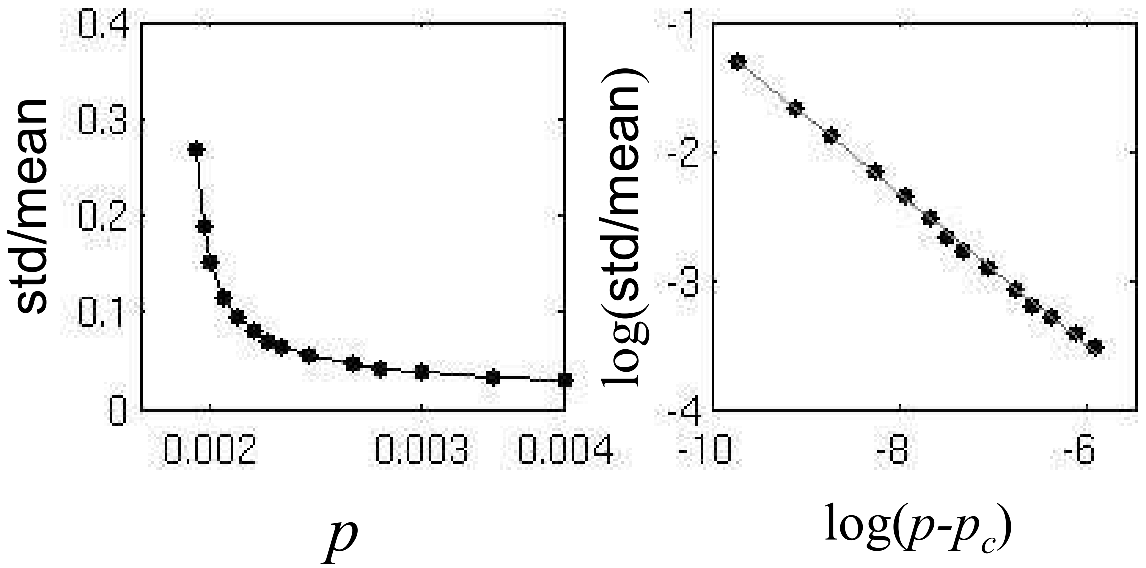
Noise-induced fluctuations

- Number of infectives fluctuates more near bifurcation point where endemic state loses stability
- Compute standard deviation of infectives over mean for long time traces
- Explained by noise-induced fluctuations near a saddle-node point

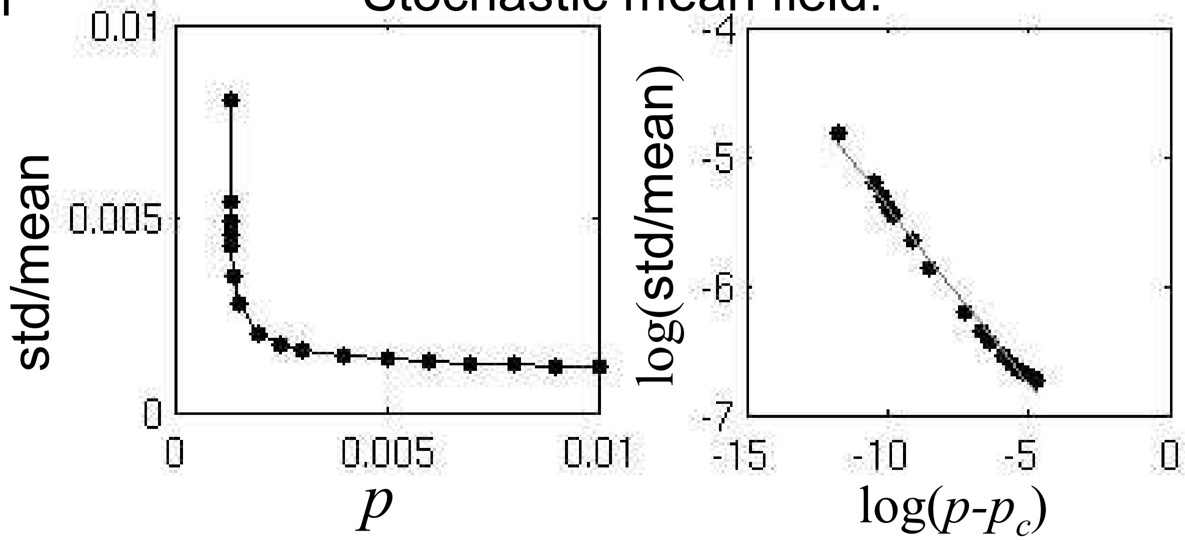


(r=0.002, w=0.04; full: q=0.0016, MF: q=0.0064)

Full system (MC):



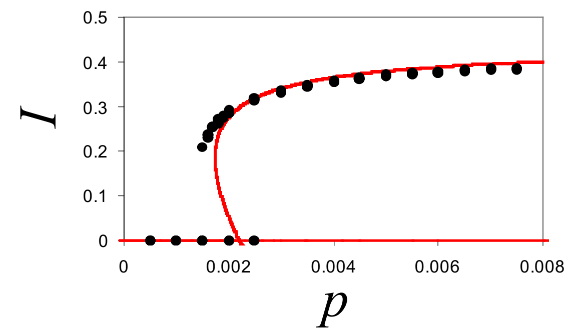
Stochastic mean field:

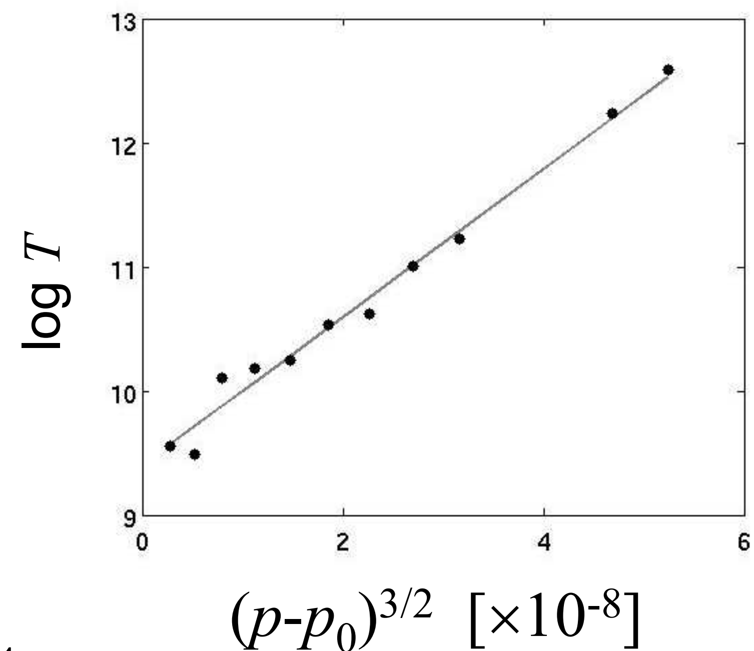


Lifetime of endemic state

- Mean lifetime T of the endemic state becomes shorter near the bifurcation point p_0 where the endemic state loses stability
- Scaling is consistent with a saddle-node bifurcation

$$\log T \sim (p - p_0)^{3/2}$$

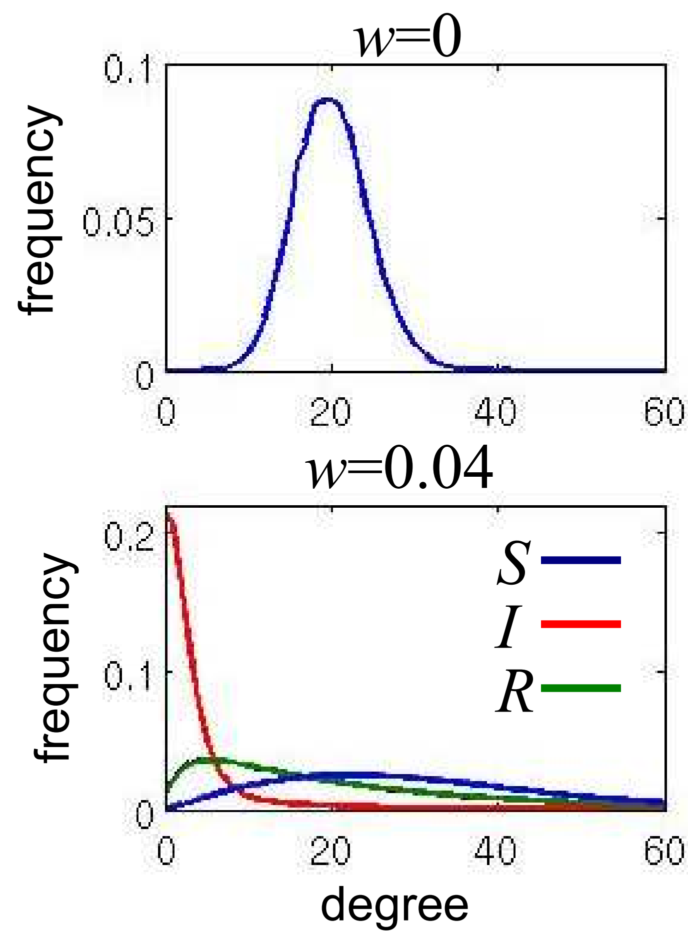




q=0.0016, r=0.002, w=0.04

Degree distributions-
Graph Topology

- Begin with Erdos-Renyi random graph (Poisson degree distribution)
- Degree distributions are significantly altered by rewiring
- Degree of infectives is reduced

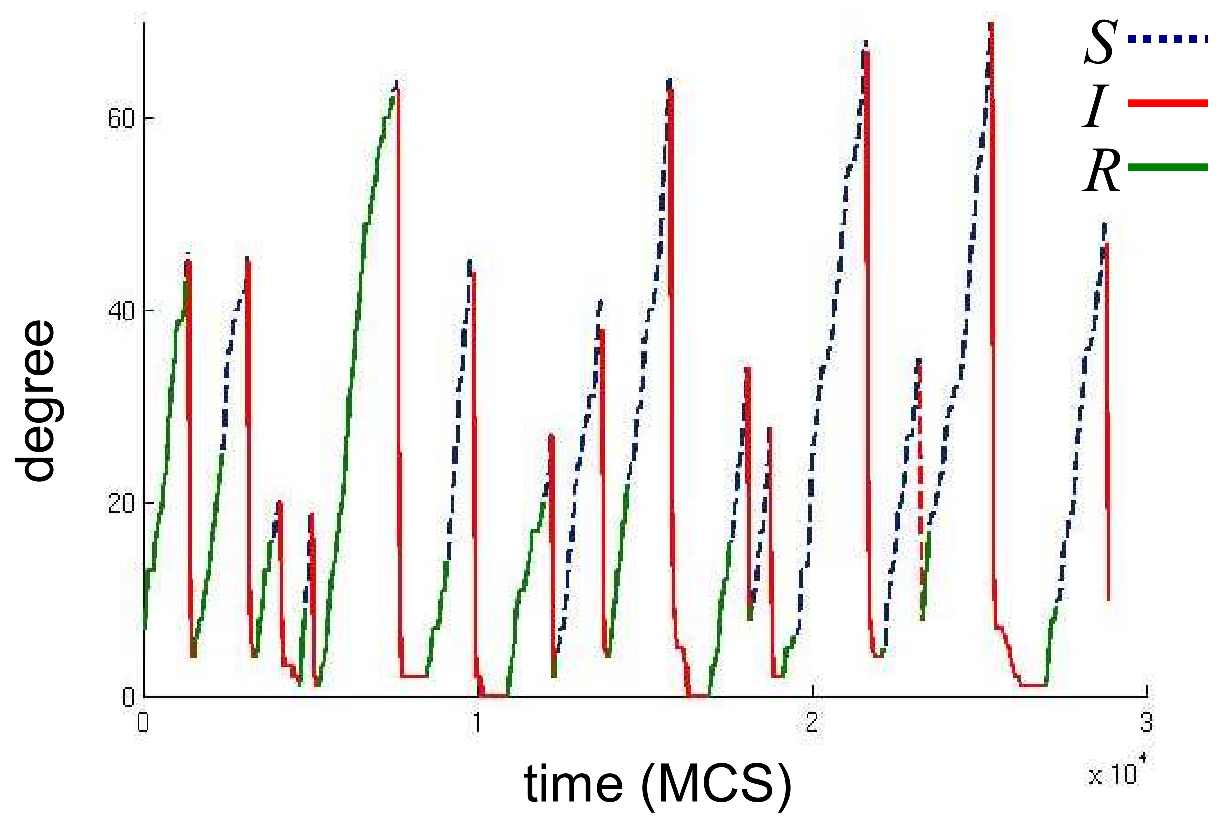


q=0.0016, r=0.002, p=0.002

Time evolution of degree

- Node degree cycles in time
- I loses links
- R and S gain links
- Examine distributions of maxima and minima in degree time series

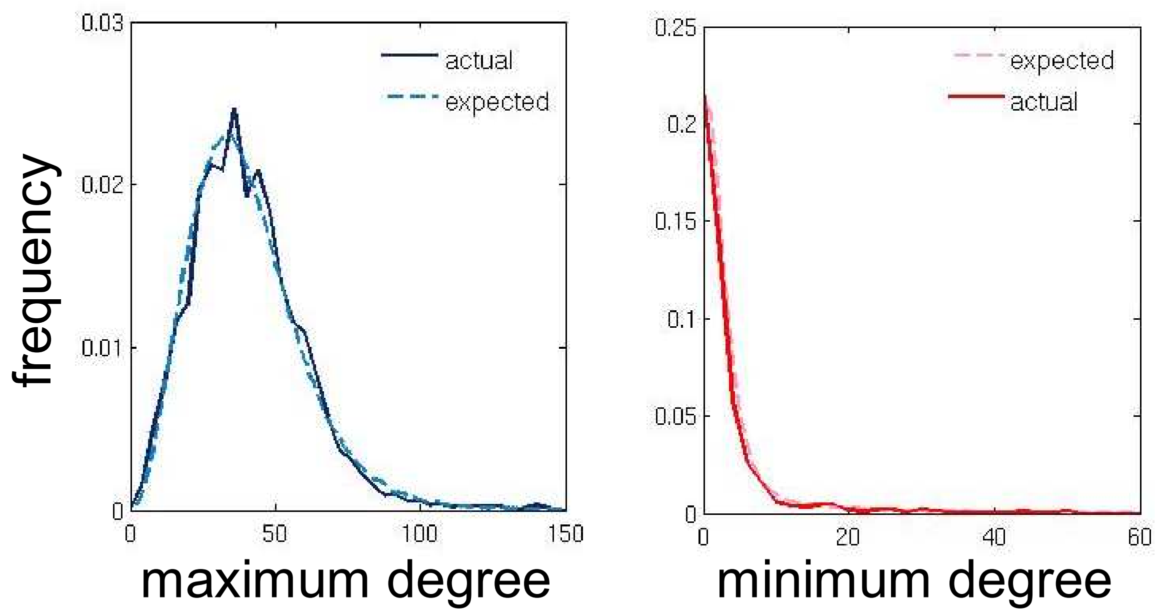
Time series for degree of a single node:



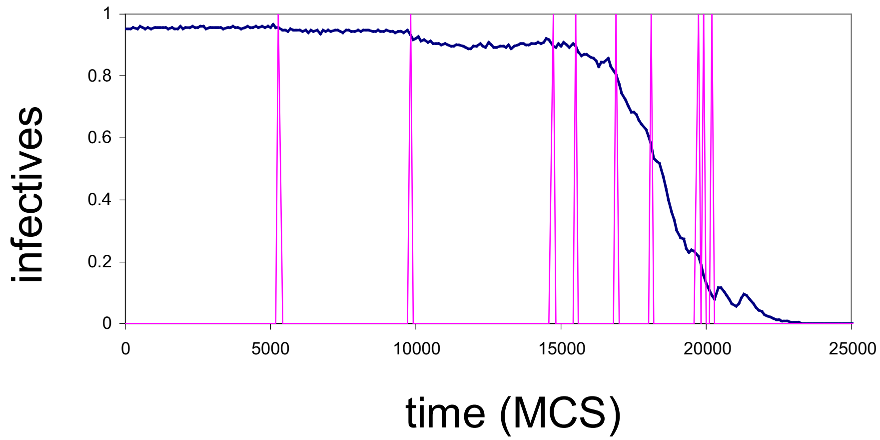
Degree maxima and minima

- Minima
  - Depend on $I \rightarrow R$
  - Recovery independent of degree so follows infective degree distribution
- Maxima
  - Depends on $S \rightarrow I$
  - Infection probabilities are proportional to degree, so distribution of maxima is degree $\times$ P(degree)

Distributions of degree maxima and minima:



Vaccine Control



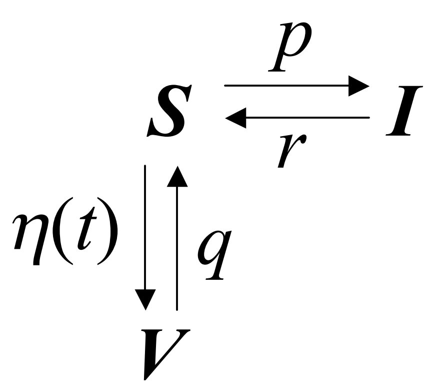
L. B. Shaw and I. B. Schwartz,
Enhanced vaccine control of epidemics in adaptive networks,
PRE in press (2010)
arXiv:0907.5539v1, (2009)

Poisson Pulse Vaccination

- Add vaccination of susceptibles
- Use vaccine pulses with Poisson-distributed schedule
  - Physical motivation: vaccine days
  - In a globally coupled system, weak Poisson-distributed vaccination leads to exponential increase in rate of infection die out (Dykman et al. PRL 101:078101, 2008; Schwartz et al., J Stat Mech 2009)
  - Non-Gaussian noise effects in networks are not well studied
- Vaccination rate is $\eta(t)$ where events happen with average frequency ν and a fraction A of susceptibles are vaccinated in each event
  - exponential distribution of times between events
  - number of events within a fixed interval is Poisson-distributed

Rules for Network Dynamics

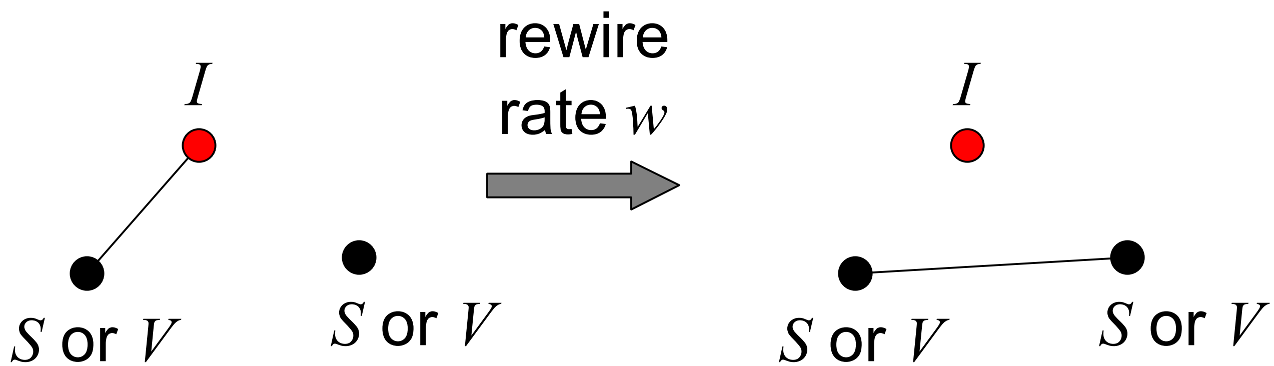
Epidemic dynamics:



S: susceptible
I: infected
V: vaccinated

p: infection rate
r: recovery rate
$\eta(t)$: vaccination rate
q: resusceptibility rate
w: rewiring rate

Network dynamics—rewiring:



Run Monte Carlo simulation for N=10^4 nodes, K=10^5 links

Mean Field Approximation

Nodes:

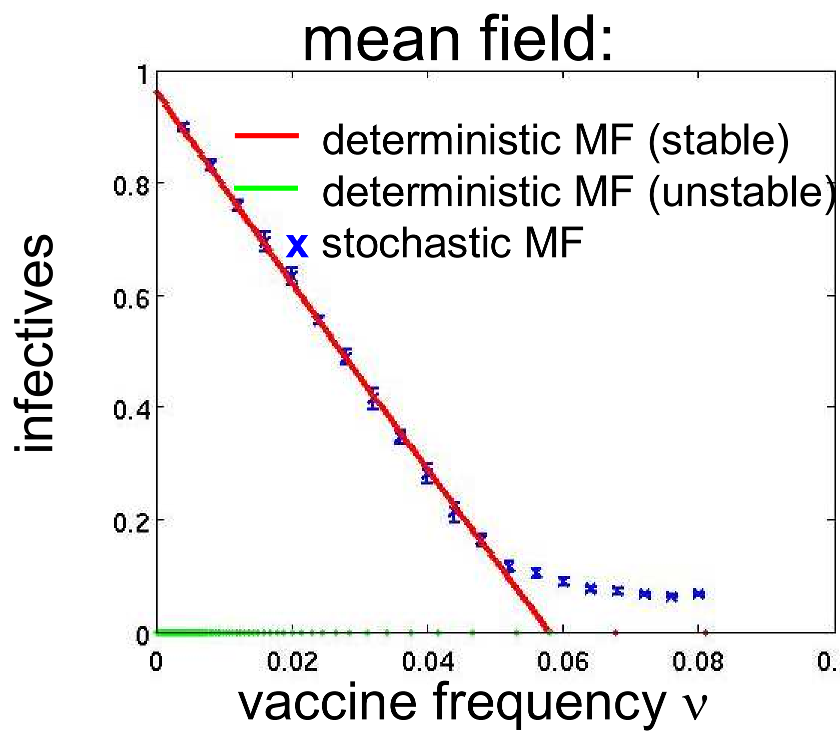
$$\begin{aligned}
\dot{P}_S &= rP_I - p\tfrac{K}{N}P_{SI} - \boxed{\eta(t)P_S} + qP_V \\
\dot{P}_I &= p\tfrac{K}{N}P_{SI} - rP_I \\
\dot{P}_V &= \boxed{\eta(t)P_S} - qP_V
\end{aligned}$$

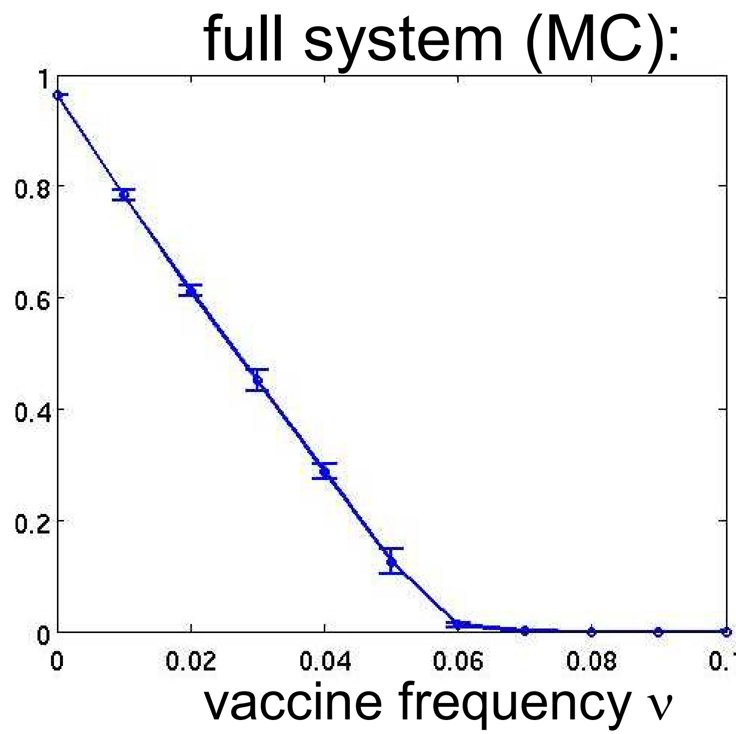
Links:

$$\begin{aligned}
\dot{P}_{SS} &= rP_{SI} - 2p\tfrac{K}{N}\tfrac{P_{SS}P_{SI}}{P_S} - 2\eta(t)P_{SS} + qP_{SV} - \eta^2(t)P_{SS} + w\tfrac{P_S}{P_S+P_V}P_{SI} \\
\dot{P}_{SI} &= 2p\tfrac{K}{N}\tfrac{P_{SS}P_{SI}}{P_S} - p\left(P_{SI} + \tfrac{K}{N}\tfrac{P_{SI}^2}{P_S}\right) - rP_{SI} + 2rP_{II} + qP_{IV} - \eta(t)P_{SI} - \boxed{wP_{SI}} \\
\dot{P}_{SV} &= rP_{IV} - p\tfrac{K}{N}\tfrac{P_{SI}P_{SV}}{P_S} + 2qP_{VV} - qP_{SV} + 2\eta(t)P_{SS} - \eta(t)P_{SV} + w\tfrac{P_V}{P_S+P_V}P_{SI} + w\tfrac{P_S}{P_S+P_V}P_{IV} \\
\dot{P}_{II} &= p\left(P_{SI} + \tfrac{K}{N}\tfrac{P_{SI}^2}{P_S}\right) - 2rP_{II} \\
\dot{P}_{IV} &= p\tfrac{K}{N}\tfrac{P_{SI}P_{SV}}{P_S} - rP_{IV} - qP_{IV} + \eta(t)P_{SI} - \boxed{wP_{IV}} \\
\dot{P}_{VV} &= \eta(t)P_{SV} - 2qP_{VV} + \eta^2(t)P_{SS} + w\tfrac{P_V}{P_S+P_V}P_{IV}
\end{aligned}$$

Vaccine effects: Static network

- Find steady state infective level as a function of vaccine frequency (for fixed amplitude)
- Mean field (deterministic and stochastic) and full system are in good agreement

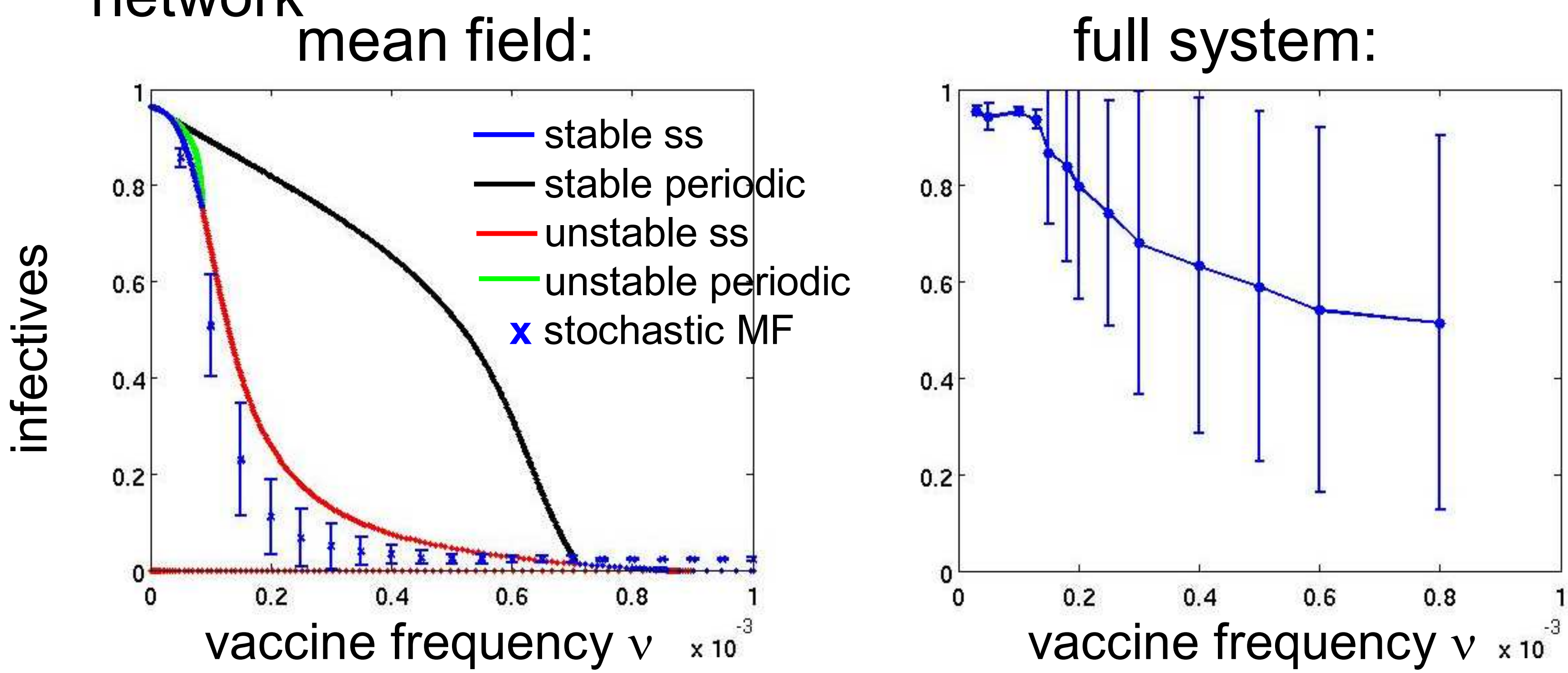




$w=0$, $p=0.003$, $r=0.002$, $q=0.0002$, $A=0.1$

Vaccine effects: Adaptive network

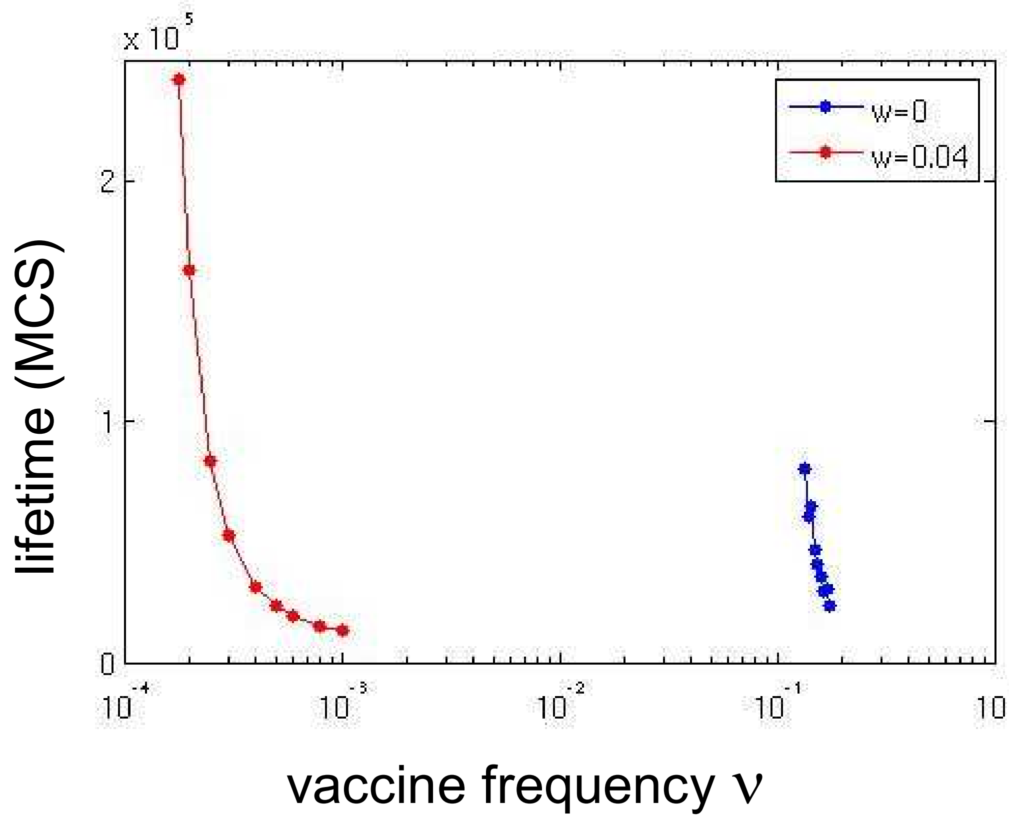
- Vaccination in an adaptive network triggers long period oscillations
- Mean field theory predicts qualitative dynamics and approximate vaccine frequency needed for eradication
- Orders of magnitude less vaccine is needed in an adaptive network



w=0.04, *p*=0.003, *r*=0.002, *q*=0.0002, *A*=0.1

Vaccine Effects: Lifetime of Endemic State

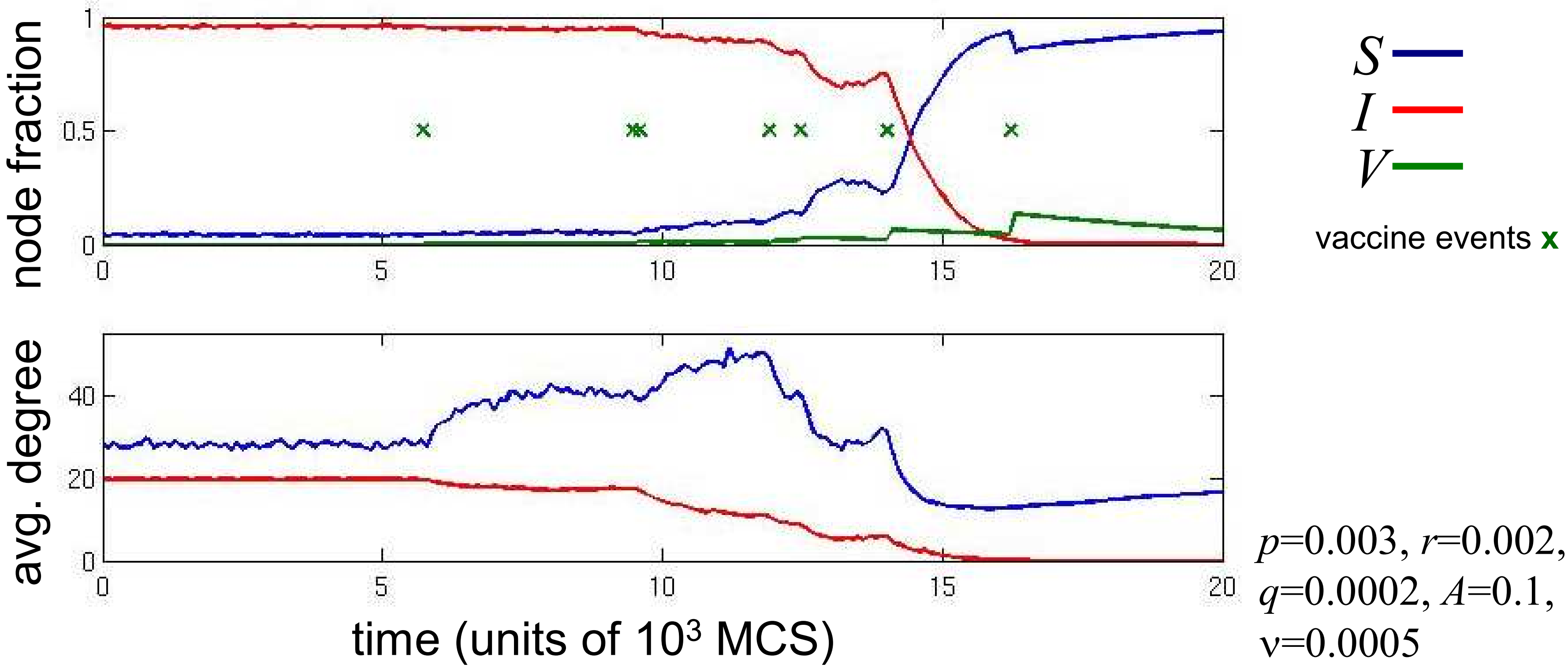
- Compute lifetime of the infected state (from the time vaccination begins)
- Average over 100 runs
- Rewiring in combination with vaccination significantly shortens the disease lifetime



p=0.003, r=0.002, q=0.0002, A=0.1

Effect of vaccination and rewiring on degree

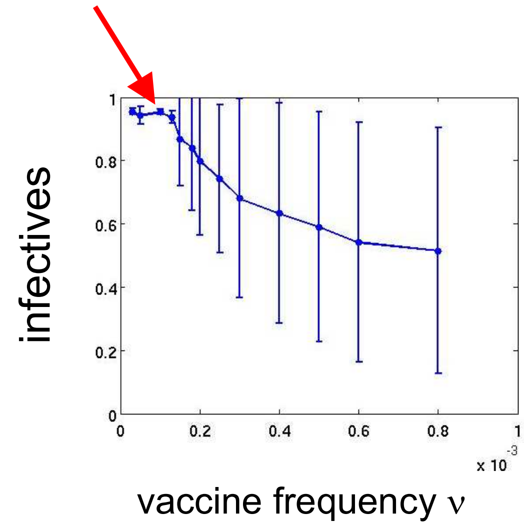
- Vaccination occurs on susceptible nodes
- In the adaptive network, susceptible nodes have higher degree due to rewiring
- Vaccination of high degree nodes provides better protection (e.g., Pastor-Satorras and Vespignani PRE 65: 036104, 2002)
- In the static network, high degree nodes tend to be infected and are not vaccinated



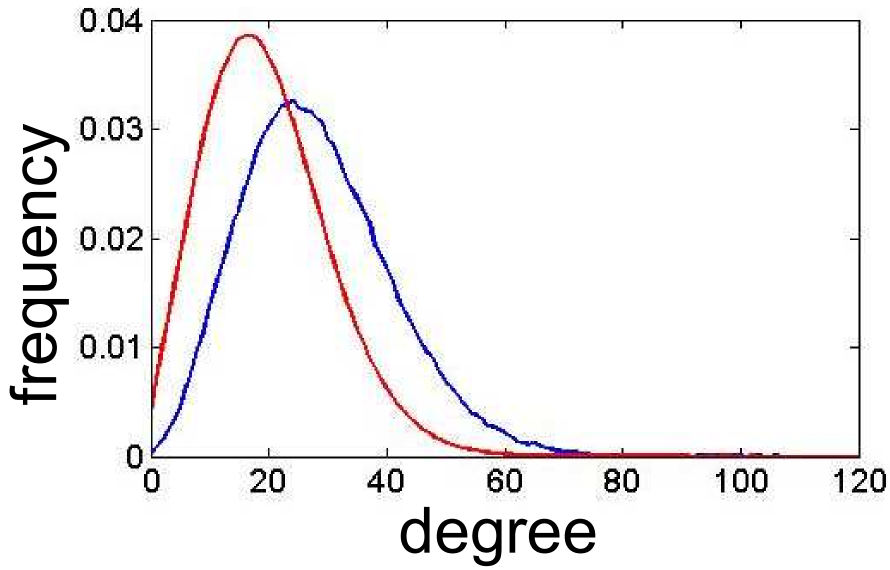
p=0.003, r=0.002, q=0.0002, A=0.1, ν=0.0005

Effect of small vaccination and rewiring on degree

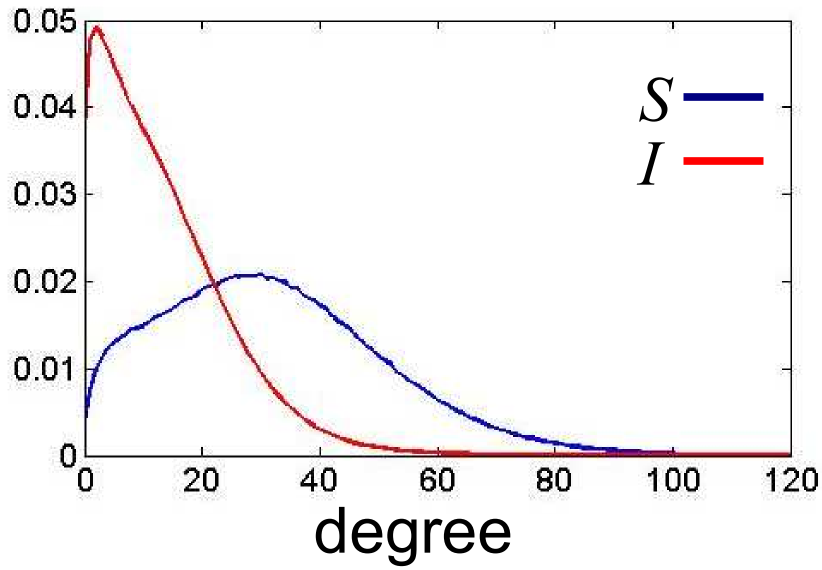
- Vaccination allows nodes to remain uninfected for longer and accumulate more neighbors
- It also provides more "safe" targets for nodes to rewire to
- Rewiring thus becomes more effective in the presence of vaccine



no vaccine:



with vaccine:



p=0.003, r=0.002, q=0.0002, w=0.04, A=0.1, ν=0 or 0.00015

Conclusions

- An epidemic model on an adaptive network was studied
  - Rewiring away from infectives leads to bistable behavior and fewer infections—predicted by lower dimensional mean field theory
  - Degree distributions altered by rewiring
  - Fluctuations and lifetimes can be understood from noise-induced dynamics near a saddle-node point
- Poisson-distributed pulsed vaccination was added
  - Vaccination and rewiring work in synergy to lower infection levels and reduce epidemic lifetimes by orders of magnitude
  - Network adaptation creates more heterogeneity in degree, and vaccination on adaptive networks exploits this heterogeneity
  - Mean field theory predicts the effect qualitatively

Future Work

- Paths to epidemic extinction in high dimensional system
- Scaling of epidemic lifetime
- Optimize control strategy
- The effect of media information on behavior
- Spatio-temporal population – community structure