Cascades in Networks and Aggregate Volatility

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Question

- Where do aggregate economic fluctuations come from?
  - Traditional answers: aggregate shocks (productivity, demand, monetary, “sunspots").
- Could aggregate fluctuations be the culmination of firm–level ups and downs?
  - Traditional answer: No. Because firm–level shocks will wash out at the rate $\sqrt{n}$ and for n large, they would be trivial.
- But this answer ignores network effects.
- When firms form networks, because they sell and purchase from each other (a supply network) or because of their financial transactions, failures of some might affect others, creating cascade effects.

Examples

"In the current crisis, we have seen that financial firms that become too interconnected to fail pose serious problems for financial stability and for regulators. Due to the complexity and interconnectivity of today's financial markets, the failure of a major counterparty has the potential to severely disrupt many other financial institutions, their customers, and other markets."

Charles Plosser
President and Chief Executive Officer,
Federal Reserve Bank of Philadelphia,
March 6, 2009

LEHMAN BROTHERS

BEAR STEARNS

"If any one of the domestic companies should fail, we believe there is a strong chance that the entire industry would face severe disruption. Ours is in some significant ways an industry that is uniquely interdependent – particularly with respect to our supply base, with more than 90 percent commonality among our suppliers. Should one of the other domestic companies declare bankruptcy, the effect on Ford's production operations would be felt within days – if not hours. Suppliers could not get financing and would stop shipments to customers. Without parts for the just-in-time inventory system, Ford plants would not be able to produce vehicles."

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This Talk

- Goal
    - A mathematical framework for evaluating how idiosyncratic shocks are translated into aggregate fluctuations because of interconnections.
- Formulation
    - A production economy with n competitive sectors
    - Explicitly model the input-output relations between sectors
    - Study the behavior of aggregate volatility (defined as the standard deviation of the aggregate output) for large n.
- Central Questions
    - Which network structures lead to higher aggregate volatilities?
    - Which are the central sectors?

Results

- Characterize the rate at which aggregate volatility vanishes for different networks
  - This rate is slower than $\sqrt{n}$ if there are important network interactions across sectors.
  - If so, sectoral shocks can translate into aggregate volatility.
- Lower bounds on the rate of decay of aggregate volatility in terms of:
  - Degrees: capturing how important the sectors are as suppliers.
  - Higher-order interconnections: capturing how connected major suppliers are through their own suppliers $\longrightarrow$ cascades.
- Preliminary empirical study of the US supply network:
  Higher-order interconnections appear to be important indeed.

Related Literature

- Study of effect of firm-level or sector-level shocks on aggregate fluctuations
  - Jovanovic (1987), Durlauf (1993): Strategic complementarities across firms
  - Back et al. (1993): Sandpile model applied to firm-level interactions
  - Gabaix (2011): Firm size distribution
- Empirical evidence on the role of sectoral shocks in macro fluctuations
  - Long and Plosser (1983), Horvath (1998, 2000), Dupor (1999), Conley and Dupor (2003) and Shea (2002)

Model: A Production Economy

- General Equilibrium (GE) framework
  - Firms
    - n competitive sectors (firms are price-takers)
    - The output of each sector is either used for production, or consumed.
    - Input-output relations explicitly modeled.
  - Consumers
    - Consume the final goods.
    - Provide labor to the market.
  - Competitive equilibrium
    - Firms and consumers behave optimally, taking the prices as given.
    - Prices are such that all markets clear.

Model:Firms

- An economy consisting of n sectors, $\mathcal{I}_n = \{1, 2, \ldots, n\}$.
- The output of each sector is used by a subset of sectors as input (intermediate goods) for production: $\mathcal{N}_i \subseteq I_n$: set of sectors that supply sector i.
- We assume that all sectors have Cobb–Douglas production technologies: output of sector i, denoted by x_i is given by

$$x_i = z_i^\alpha l_i^\alpha \prod_{j \in \mathcal{N}_i} x_{ij}^{(1-\alpha) w_{ij}},$$

  where

  - l_i: labor employed by sector i,
  - $\alpha \in (0, 1]$: labor share in the production technologies,
  - x_{ij} : amount of commodity j used in the production of good i,
  - $w_{ij} \geqslant 0$: input share of sector j in sector i's production. The supply relations is captured by the input-output matrix $W_n = [w_{ij}]_{i,j \in \mathcal{I}_n}$.
  - z_i : productivity shock to sector i. We assume that z_i are independent across sectors, and denote the distribution of $\epsilon_i = \log(z_i)$ by F_i.
- We use $\mathcal{E}_n = (I_n, W_n, \{F_i\}_{i \in \mathcal{I}_n})$ to refer to an economy of size n.

Assumptions

Assumption

The input shares of any firm $i \in \mathcal{I}_n$ in the economy add up to one, i.e., $\sum_{j=1}^{n} w_{ij} = 1$.

- This assumption guarantees that the production functions exhibit constant returns to scale to their labor inputs and intermediate goods provided by suppliers.
- It guarantees that the input-output matrix W_n is a (row) stochastic matrix, i.e., all rows add up to one.

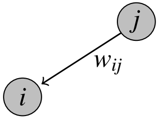
Assumption

Given a sequence of economies $\{\mathcal{E}_n\}_{n \in \mathbb{N}}$ and for any sector $i \in \mathcal{I}_n$, F_i is such that

1. $\mathbb{E}\epsilon_i = 0$, *and*
2. $\text{var}(\epsilon_i) = \sigma_i^2 \in (\underline{\sigma}^2, \bar{\sigma}^2)$, *where $0 < \underline{\sigma} < \bar{\sigma}$ are independent of n.*

Model: Supply Network

$$x_i = z_i^{\alpha} l_i^{\alpha} \prod_{j \in \mathcal{N}_i} x_{ij}^{(1-\alpha) w_{ij}},$$



- Supply network: weighted, directed graph $G_n = (\mathcal{I}_n, E_n, W_n)$
- Degree of sector j: share of sector j's output in the input supply of the economy, $d_j = \sum_{i=1}^{n} w_{ij}$

- Representative firm in sector i solves the problem:

$$\max_{l_i, x_i, \{x_{ij}\}_{j \in \mathcal{I}_n}} \quad p_i x_i - h l_i - \sum_{j=1}^{n} p_j x_{ij}$$

$$\text{subject to} \quad x_i = e^{\alpha \epsilon_i} l_i^{\alpha} \prod_{j=1}^{n} x_{ij}^{(1-\alpha) w_{ij}}.$$

  - h is the market wage, p_i is the market price of good i.

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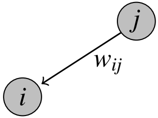

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Model:Consumers

- A continuum of identical consumers of mass one.
- Representative household endowed with one unit of labor.
- Preferences over all goods in the economy are given by

$$u(c_1, c_2, \ldots, c_n) = A_n \prod_{i=1}^{n} (c_i)^{1/n}.$$

- Representative consumer's problem:

$$\begin{aligned} \max_{\{c_i\}_{i \in \mathcal{I}_n}} \quad & u(c_1, \ldots, c_n) \\ \text{subject to} \quad & p_1 c_1 + \cdots + p_n c_n = h \end{aligned}$$

Competitive Markets Equilibrium

Definition

In the competitive markets equilibrium of economy, the prices $(p_1, p_2, \ldots, p_n)$ and wage h are such that

(a) the representative consumer maximizes her utility,

(b) the representative firms in each sector maximize profits,

(c) labor and commodity markets clear.

$$
\begin{aligned}
c_i^* + \sum_{j=1}^{n} x_{ji}^* &= x_i^* \qquad \forall i \in \mathcal{I}_n \\
\sum_{i=1}^{n} l_i^* &= 1.
\end{aligned}
$$

Competitive Equilibrium

- The aggregate value added in the economy is given by the equilibrium wage h.
- We define the aggregate output y_n as

$$y_n \equiv \log(h).$$

Proposition

At the equilibrium, the aggregate output is a convex combination of the sectoral shocks:

$$y_n = \sum_{i=1}^{n} v_{n,i}\epsilon_i = v_n'\epsilon,$$

where $\epsilon = [\epsilon_1, \ldots, \epsilon_n]$ *and* v_n *is the influence vector given by*

$$v_n \equiv \frac{\alpha}{n}\left[I - (1-\alpha)W_n'\right]^{-1}\mathbf{1}.$$

The Influence Vector

- Since none of W_n's eigenvalues lie outside of the unit circle, it is possible to express v_n in terms of a convergent power series:

$$v_n' = \frac{\alpha}{n}\mathbf{1}' \sum_{k=0}^{\infty} (1-\alpha)^k W_n^k.$$

- Or alternatively as:

$$v_n' = \frac{\alpha}{n}\mathbf{1}' + (1-\alpha) v_n' W_n.$$

- The presence of higher-order interconnections can already be seen from this equation (the second term on the right-hand side).
- Note that the influence vector related to the definition of the PageRank vector or the Bonacich centrality in the Internet search algorithms.

Alternative Interpretations

- We could have alternatively consider a reduced-form model

$$\tilde{y} = (1-\alpha)\tilde{W}_n\tilde{y} + \alpha\tilde{\epsilon}.$$

- This could arise, for example, from:

1. Models in which ϵ_i's are not productivity shocks, but other shocks to sectoral or firm behavior.
2. Models in which units are firms rather than sectors (but then one needs to model "relationship-specific investments" and to some degree endogenize $\tilde{W}_n$).
3. Financial models with counterparty relationships between financial institutions. In this case, $w_{ij} > 0$ would correspond to firm i being a counterparty to firm j (i.e., holding some of firm j's debt or other liabilities on its balance sheet).
4. Models of "strategic complementarities".

Aggregate Volatility

- Recall aggregate output is given by $y_n = v_n' \epsilon$.
- We focus on aggregate volatility, defined as the standard deviation of aggregate output.
- Aggregate volatility is clearly equal to:

$$\sigma_{\text{agg}} \equiv (\text{var}\, y_n)^{1/2} = \sqrt{\sum_{i=1}^{n} \sigma_i^2 v_{n,i}^2}.$$

- Since σ_i is uniformly bounded, this implies that aggregate volatility scales with $\|v_n\|_2$, where $\|\cdot\|_2$ is the Euclidean (vector) norm.
- That is:

$$\sigma_{\text{agg}} = \Theta(\|v_n\|_2).$$

$a_n = \Theta(b_n) \iff 0 < \liminf_{n\to\infty} a_n/b_n \leq \limsup_{n\to\infty} a_n/b_n < \infty.$

Dominant Sectors

Definition

A sequence of economies $\{\mathcal{E}_n\}_{n\in\mathbb{N}}$ has dominant sectors if $\|v_n\|_\infty = \Theta(1)$, where $\|\cdot\|_\infty$ refers to the sup (vector) norm, i.e., the largest element of the vector.

- As $n \to \infty$, the economy has more and more sectors subject to independent shocks. If these sectors are all "symmetric," the influence of each sector should gradually decline, i.e., $\|v_n\|_\infty \to 0$ (which also implies $\|v_n\|_2 \to 0$).
- We would have $\|v_n\|_\infty \not\to 0$ only if the influence of some sectors does not die down even as the economy becomes very large.

Definition

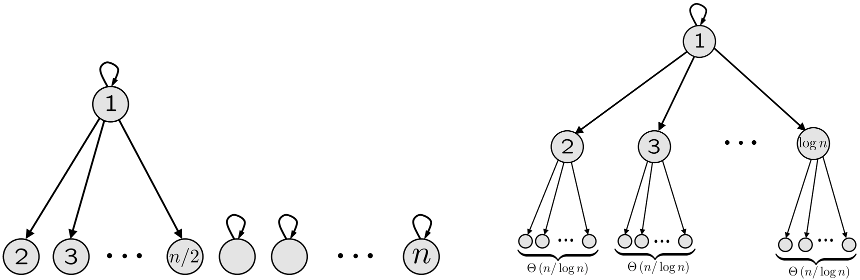
A sequence of economies $\{\mathcal{E}_n\}_{n\in\mathbb{N}}$ has a star-like structure if $\max_{i\in\mathcal{I}_n} d_i = \Theta(n)$, where d_i denotes the degree of sector i.

Star-like Structures and Beyond

Proposition

A sequence of economies $\{\mathcal{E}_n\}_{n\in\mathbb{N}}$ *with a star-like structure has dominant sectors.*

- The main intuition is that idiosyncratic shocks to dominant firms propagate through the economy and affect the outputs of a non–vanishing fraction of firms.



The Law of Large Numbers in Networks

Theorem

Aggregate output in a sequence of economies $\{\mathcal{E}_n\}_{n\in\mathbb{N}}$ converges to zero in probability (as $n \to \infty$) if and only if $\{\mathcal{E}_n\}_{n\in\mathbb{N}}$ does not have dominant sectors.

- Important, but somewhat extreme. In most realistic economies, we would expect the law of large numbers to hold, but network effects to be still present.
- This is the main focus of our investigation.

First-Order Interconnections

- How is $\|v_n\|_2$ related to the structural properties of the network?
  - Lower bounds on the rate of decay of aggregate volatility in terms of structural properties of the supply network.
- We first provide lower bounds obtained from first-order interconnections, represented by the degree distribution.

Definition

Given an economy $\mathcal{E}_n$ with degree sequence $d^{(n)} = (d_1, d_2, \ldots, d_n)$, the coefficient of variation is

$$\mathrm{CV}(d^{(n)}) \equiv \frac{\mathrm{STD}(d^{(n)})}{\bar{d}}$$

where $\bar{d} \equiv \frac{1}{n}\sum_{i=1}^{n} d_i$ is the average degree, and $\mathrm{STD}(d^{(n)}) \equiv \left[\sum_{i=1}^{n}(d_i - \bar{d})^2/(n-1)\right]^{1/2}$ is the standard deviation of the degree sequence.

First-Order Interconnections and Aggregate Volatility

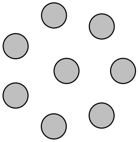
Theorem

For any sequence of economies, aggregate volatility satisfies

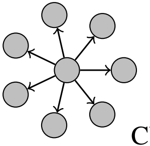
$$\sigma_{\text{agg}} = \Omega\left(\frac{1 + \mathrm{CV}(d^{(n)})}{\sqrt{n}}\right).$$

$a_n = \Omega(b_n) \iff \liminf_{n\to\infty} a_n/b_n > 0.$

- High variability in the out-degrees implies slower rates of decay and thus, higher levels of aggregate volatility.



$\mathrm{CV} = 0.$



$\mathrm{CV} \sim \sqrt{n}$

Power Law Degree Distributions and Aggregate Volatility

- Economies with power law tails: $1 - \text{cdf} \sim d^{-\beta}$.
  - Lower values of β correspond to heavier tails.

Corollary

If the degree distribution of a sequence of economies has a power law tail with shape parameter $\beta \in (1,2)$, then aggregate volatility satisfies

$$\sigma_{\text{agg}} = \Omega\left(n^{-\frac{\beta-1}{\beta}-\epsilon}\right),$$

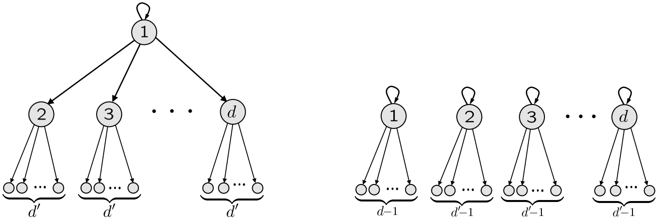
where $\epsilon > 0$ is arbitrary.

- A smaller β corresponds to higher aggregate fluctuations.

Higher-Order Interconnections and Cascades

- The degree distribution only captures first-order interconnections.
- Cascades are instead about higher-order interconnections.
- The degree distribution provides little information about higher-order interconnections.

Example: The two economies $\{\mathcal{E}_n\}_{n\in\mathbb{N}}$ and $\{\widehat{\mathcal{E}}_n\}_{n\in\mathbb{N}}$ depicted below have identical degree sequences. However, it can be verified that $\|v\|_2 = \Theta(1)$ whereas $\|\widehat{v}\|_2 = \Theta(d/n + 1/\sqrt{d})$ (e.g., when d is of order $\Theta(\sqrt{n})$, then $\|\widehat{v}\|_2 = \Theta(1/\sqrt[4]{n})$).



Second-Order Interconnections

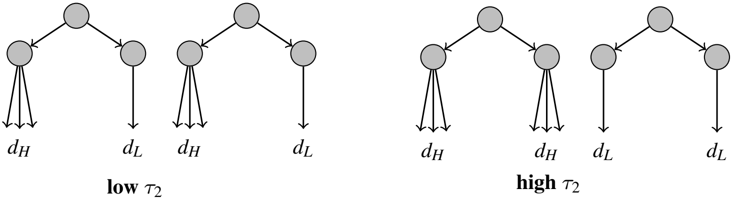
Definition

The second-order interconnectivity coefficient is defined as

$$\tau_2(W_n) \equiv \sum_{i=1}^{n} \sum_{j \neq i} \sum_{k \neq i,j} w_{ji} w_{ki} d_j d_k,$$

where d_j is the degree of sector j.

- τ_2 takes higher values when high degree sectors share the same suppliers with other high-degree sectors $\rightarrow$ opening the way to cascades.

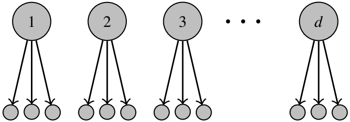


Second-Order Interconnections and Cascades

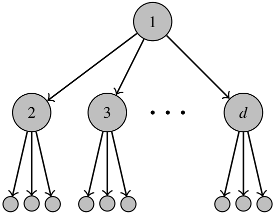
Theorem

Given a sequence of economies, the aggregate volatility satisfies

$$\sigma_{\text{agg}} = \Omega\left(\frac{1}{\sqrt{n}} + \frac{\text{CV}}{\sqrt{n}} + \frac{\sqrt{\tau_2(W_n)}}{n}\right)$$



$\tau_2 = \mathbf{0}$



$\tau_2 \sim \mathbf{n^2}$

Power Law Distribution of Second-Order Degrees

- Second-order degrees:

$$q_i \equiv \sum_{j=1}^{n} d_j w_{ji}.$$

Corollary

If the second-order degrees of a sequence of economies have a power law tail with shape parameter $\zeta \in (1,2)$, then aggregate volatility satisfies

$$\sigma_{\text{agg}} = \Omega\left(n^{-\frac{\zeta-1}{\zeta}-\epsilon}\right),$$

for any $\epsilon > 0$.

- If both the first-order and second-order degrees have power law tails:

$$\sigma_{\text{agg}} = \Omega\left(n^{-\frac{\beta-1}{\beta}} + n^{-\frac{\zeta-1}{\zeta}}\right)$$

- Dominant term: $\min\{\beta, \zeta\}$.

Higher-Order Degrees and Cascades

Definition

Given an economy, the $(m+1)^{th}$-order interconnectivity coefficient is defined as

$$\tau_{m+1}(W_n) \equiv \sum_{i=1}^{n} \sum_{\substack{j_1,\ldots,j_m \\ k_1,\ldots,k_m \\ \text{all distinct}}} (d_{j_1} d_{k_1}) \, (w_{j_m i} w_{k_m i}) \prod_{s=1}^{m-1} w_{j_s j_{s+1}} \prod_{r=1}^{m-1} w_{j_r j_{r+1}}$$

- τ_m captures the extent to which large suppliers share suppliers m levels upstreams in the chain.

Theorem

Given a sequence of economies and for any $m \in \mathbb{N}$, aggregate volatility satisfies

$$\sigma_{\text{agg}} = \Omega\left(\frac{1}{\sqrt{n}} + \frac{\text{CV}}{\sqrt{n}} + \frac{\sqrt{\tau_2(W_n)}}{n} + \cdots + \frac{\sqrt{\tau_m(W_n)}}{n}\right).$$

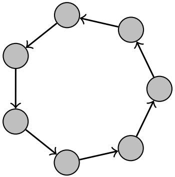
When Higher-Order Interconnections Do Not Matter

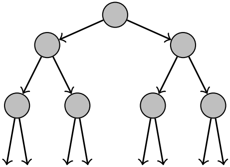
Definition

A sequence of economies is balanced if $\max_i d_i < c$ for some positive constant c and all n.

Proposition

For any sequence of balanced economies, $\sigma_{agg} \sim 1/\sqrt{n}$.





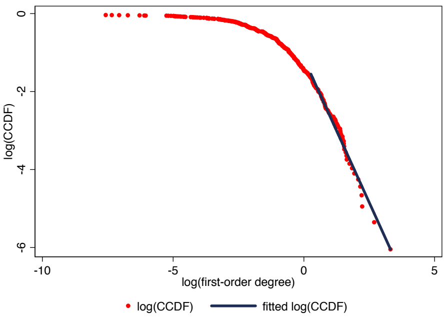
Application: The US Supply Network

- The US input-output matrix (not disaggregated enough, but still useful).
- 423 sectors in the commodity-by-commodity direct requirements table. (Bureau of Economic Analysis)
- This gives us the equivalent of our W_n matrix.
- Includes sectors
  - Semi-conductor and related device manufacturing, Wholesale trade, Retail trade, Real estate, Truck transportation, Advertising and related services.

First and Second-Order Degree Distributions

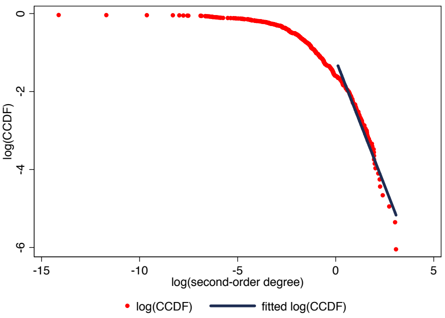
- Linear tail in the log-log scale (20% of the observations = 82 industries).
- Power law is a fairly good approximation to the tail of the distribution.

First-order degree distribution



$\beta \approx 1.5$

Second-order degree distribution



$\zeta \approx 1.3$

Implied Behavior of Aggregate Volatility

- $\zeta < \beta$: second-order effects dominate first-order effects.
- Average (annual) standard deviation of the logarithm of value-added across 459 four-digit (SIC) manufacturing industries between 1958 and 2005 is 0.219.
  (NBER Manufacturing Productivity Database)
- Since manufacturing is about 20% of the economy, for the entire economy this corresponds to $5 \times 459 = 2295$ sectors at a comparable level of disaggregation.
- Had the structure been balanced: $\sigma_{\text{agg}} = 0.219/\sqrt{2295} \simeq 0.005$.
- But from the lower bound from the second-order degree distribution:

$$\sigma_{\text{agg}} \sim \sigma / n^{0.23} \simeq 0.037$$

Summary

- A microfounded model to explain how idiosyncratic shocks can lead to aggregate fluctuations.
- A characterization of the relation between the network structure and aggregate volatility.
- The importance of higher order interconnections, both in theory and practice.
- Characterization of tail events as a function of network structure.

Future Work:

- Endogenize network structure.
- Dynamic network linkages and propagation of volatility over time.
- Cascades in financial networks.
- Firm-level instead of sectoral investigation:
  - Market power.
  - Upstream as well as downstream propagation.
- Systematic empirical investigation.