Connectivity and Resilience of Large-scale Wireless Networks: A Percolation View

Edmund M. Yeh

Department of Electrical and Computer Engineering

Northeastern University

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Large-scale Networks

- This is the era of networks:
  - Physical: power grids and roads
  - Biological: neurons and genomic
  - Social: Facebook
  - Information: Internet, sensor networks
- Network science is an effort to characterize common structures and properties in all these networks.

Large-scale Networks

National Research Council report (05) major research challenges:

- Modeling and analysis of very large networks
- Increasing level of rigor and mathematical structure
- Abstracting common concepts across fields
- Dynamics, spatial location, and information propagation in networks
- Robustness of security of networks

Our Focus

- Main characteristics:
  - Large scale
  - Spatial location and geometry
  - Mobility
- Main issues:
  - Connectivity
  - Information dissemination
  - Network resilience and security

Connectivity, Epidemics, and Resilience

1. Unifying concepts: connectivity and percolation theory
2. Information/virus spread in wireless networks with dynamic links
3. Information/virus spread in mobile wireless networks
4. Viral epidemics and cascading failures

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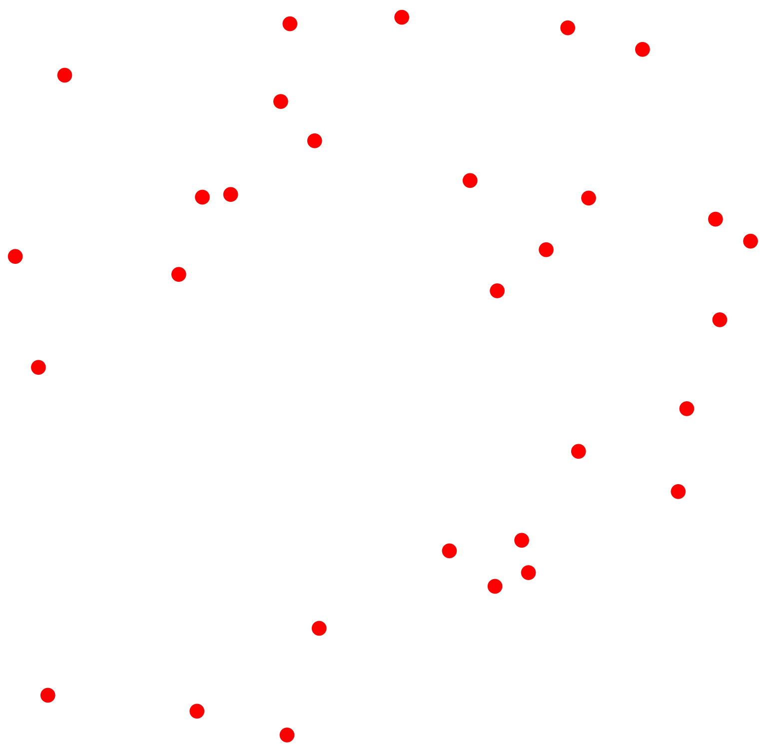
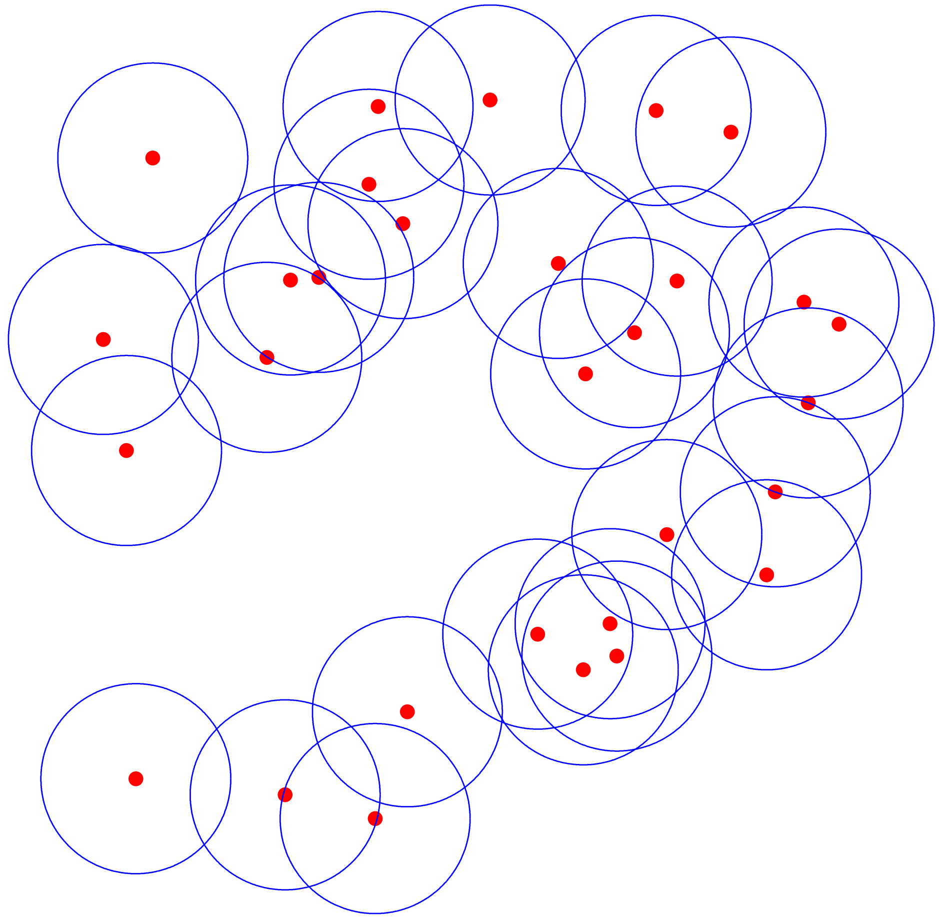
Continuum Percolation



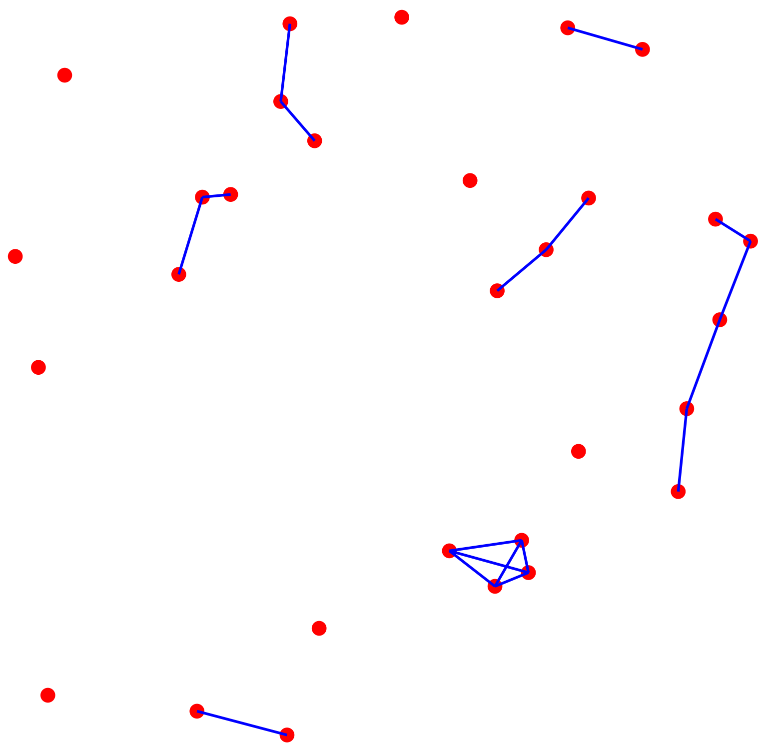
- Set of nodes uniformly distributed at random over an area

Continuum Percolation



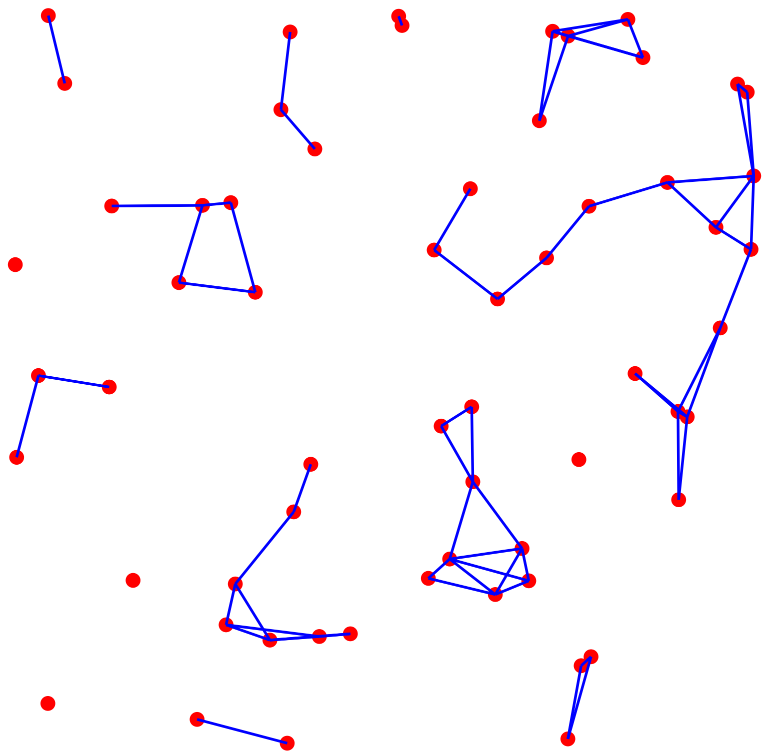
- Each node has communication radius = 1.

Continuum Percolation



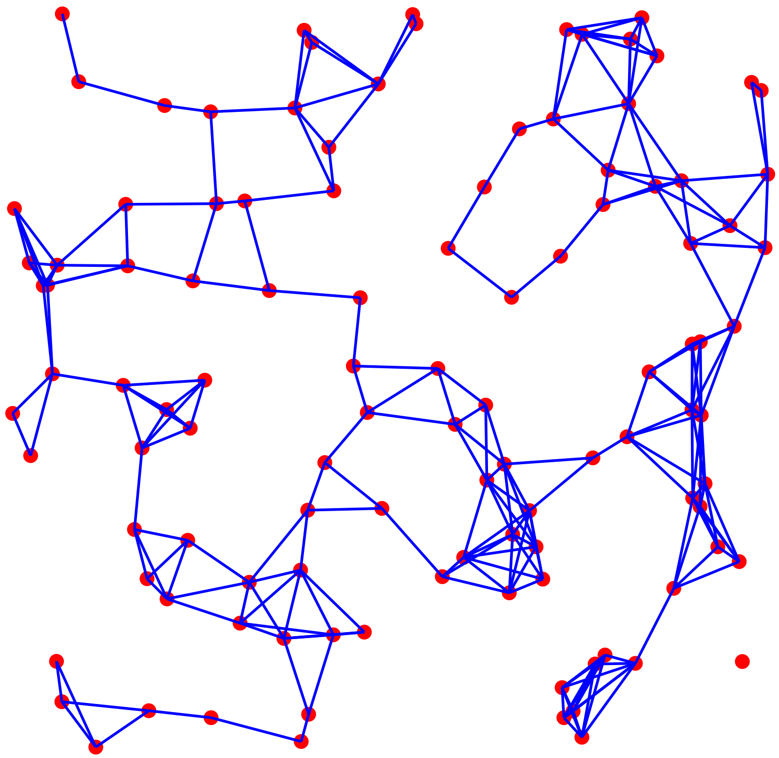
- Place link between nodes that can directly communicate.

Continuum Percolation



- As density increases, small connected clusters emerge, but largest component has $O(\log n)$ nodes.

Phase Transition and Giant Component



- At critical point, giant component spanning whole network ($\Theta(n)$ nodes) forms.

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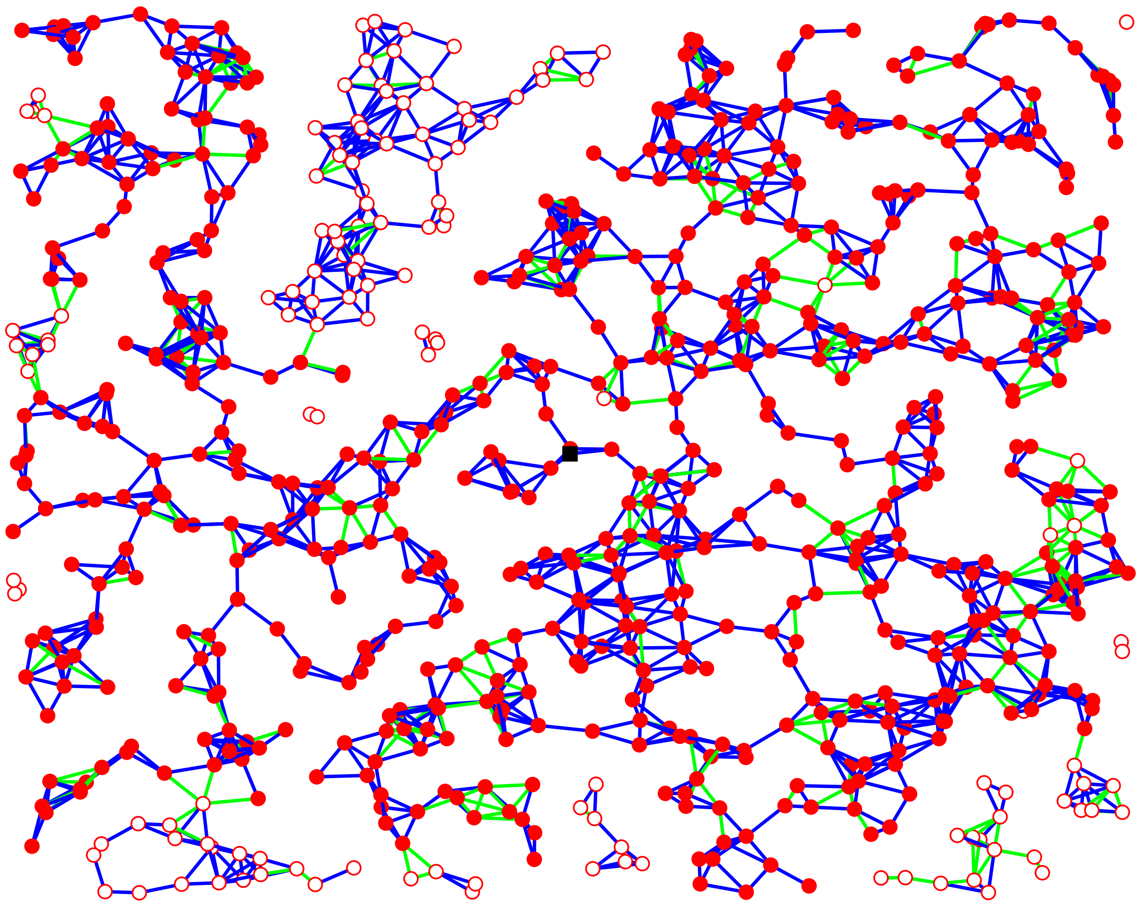
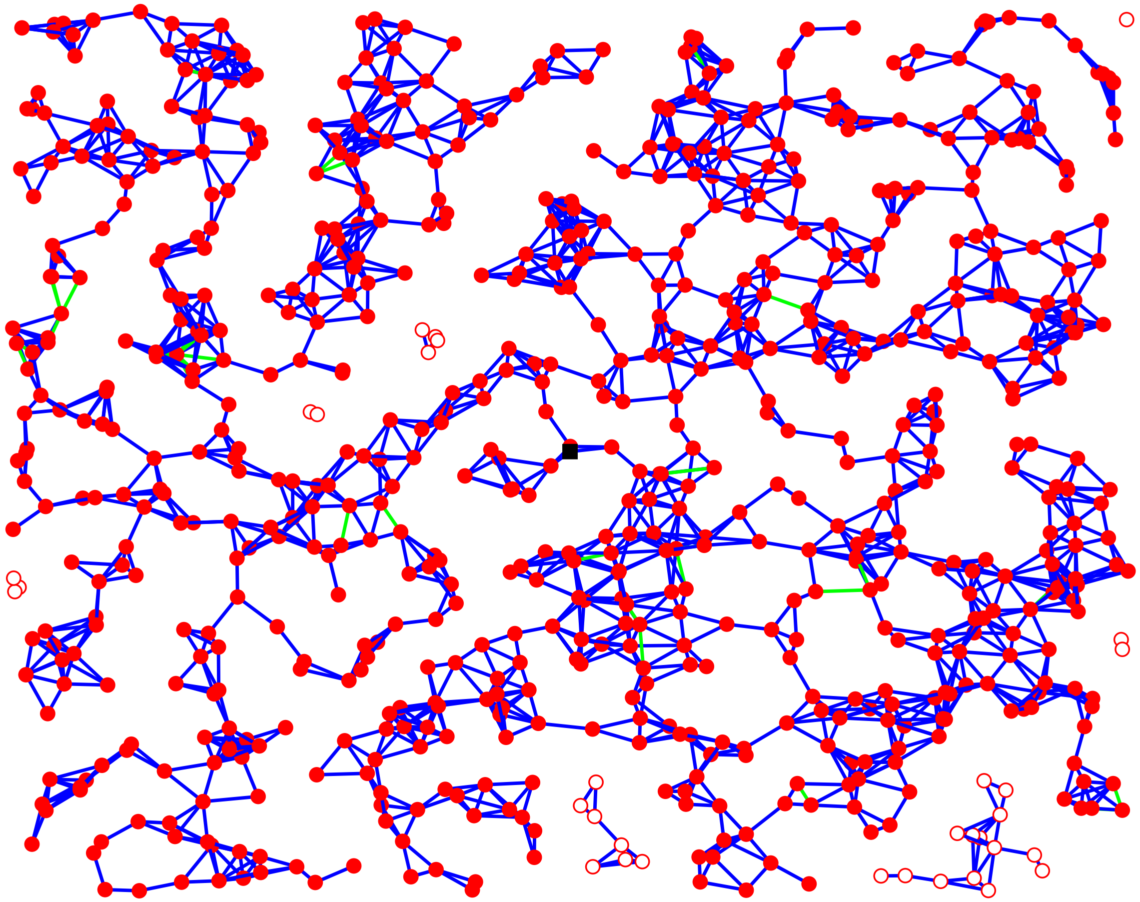
Information/Virus Spread: Supercritical Phase



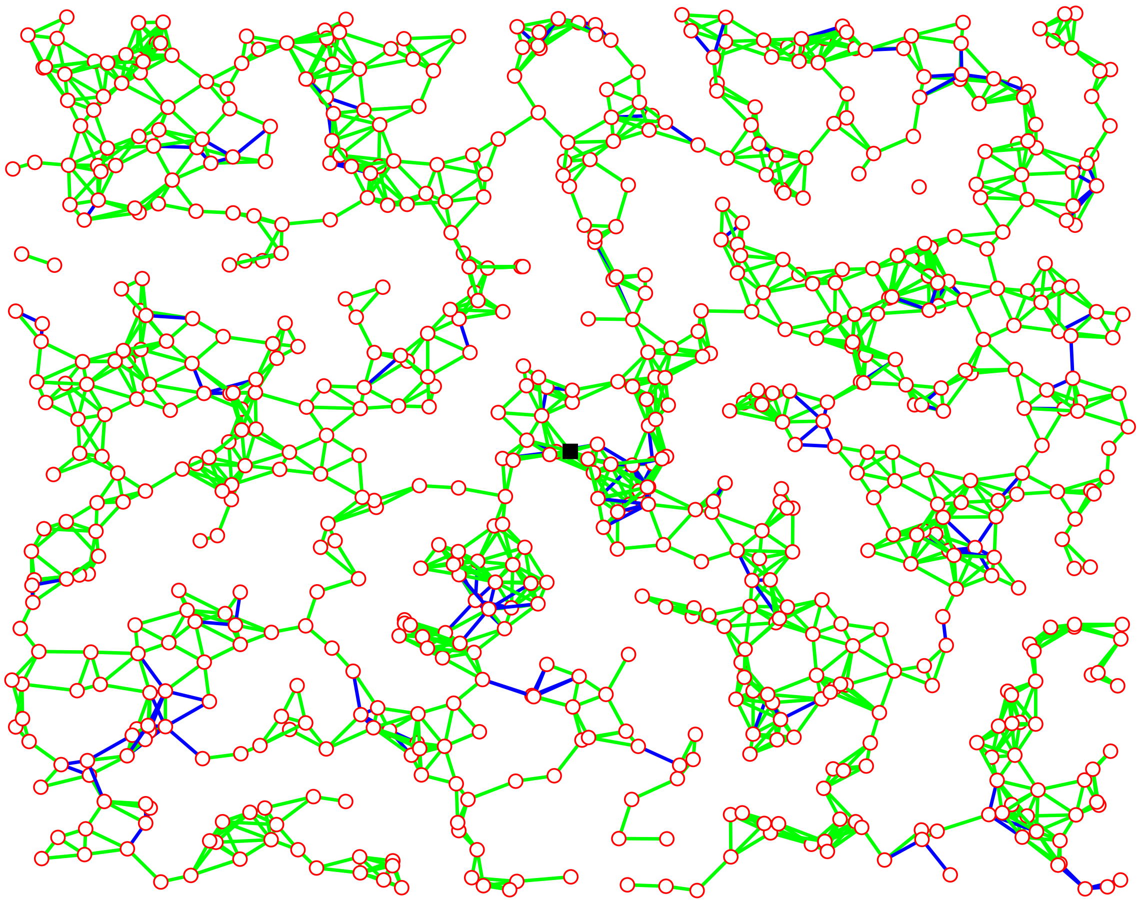
- 750 nodes on $[-10, 10]^2$, $E[Z] = 2$, $E[Y] = 0.5$.
- Source (black nodes) broadcasts message M at $t = 0$.
- Links: blue—on, green—off; Nodes: red—received, white—not received

Information/Virus Spread: Supercritical Phase



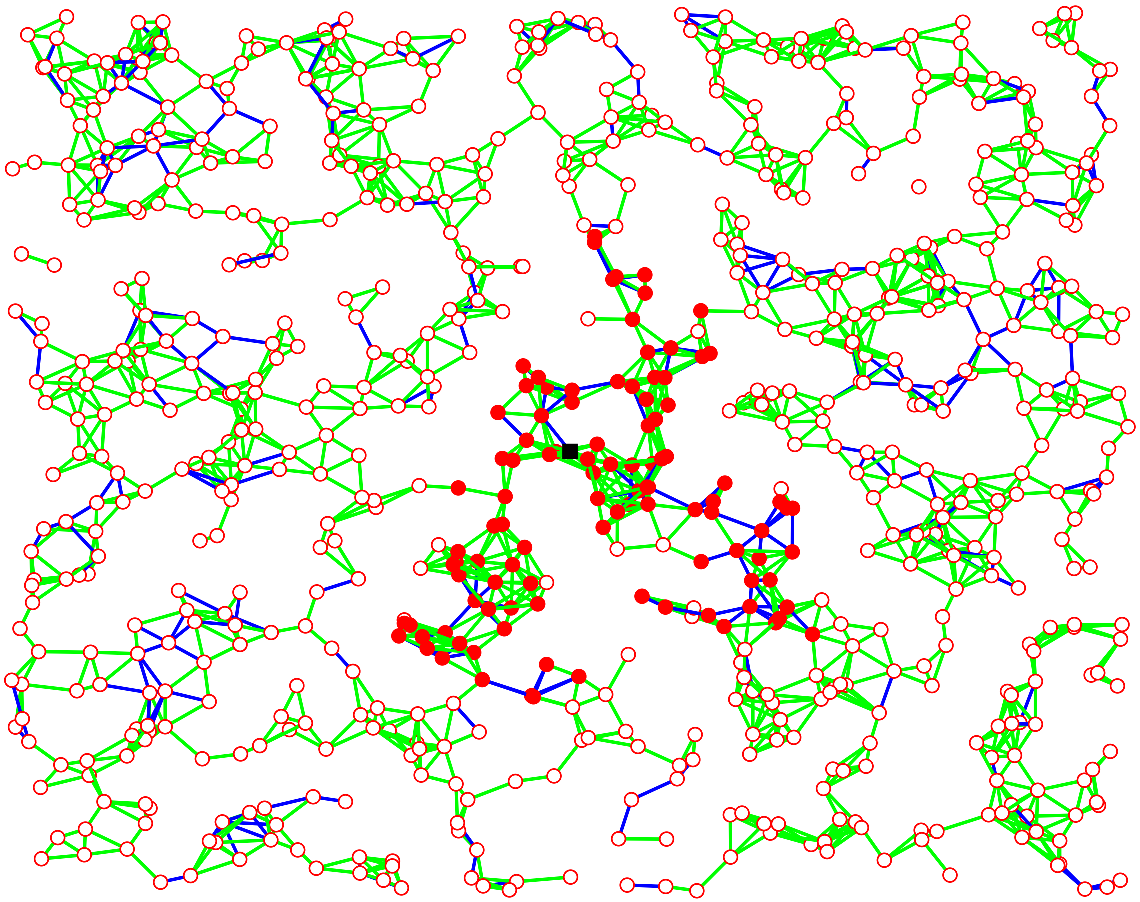
- 750 nodes on $[-10, 10]^2$, $E[Z] = 2$, $E[Y] = 0.5$.
- By $t = 0.1021$, all nodes in $\mathcal{C}(G(\mathcal{H}_\lambda, 1))$ have received M.
- Links: blue—on, green—off; Nodes: red—received, white—not received

Information/Virus Spread: Subcritical Phase



- 750 nodes on $[-10, 10]^2$, $E[Z] = 0.25$, $E[Y] = 2$.
- Source (black nodes) broadcasts message M at $t = 0$.
- Links: blue—on, green—off; Nodes: red—received, white—not received

Information/Virus Spread: Subcritical Phase



- 750 nodes on $[-10, 10]^2$, $E[Z] = 0.25$, $E[Y] = 2$.
- By $t = 1.0628$, only 102 nodes have received message M.
- Links: blue—on, green—off; Nodes: red—received, white—not received

Delay Scaling: Two Regimes

Theorem: Given percolated random geometric graph $G(\mathcal{H}_\lambda, 1)$:

(i) if $G(\mathcal{H}_\lambda, 1, W(t))$ is not percolated at any time, then for $d(u,v)$ large,

$$\frac{T(u,v)}{d(u,v)} \sim \gamma$$

with high probability.

(ii) if $G(\mathcal{H}_\lambda, 1, W(d,t))$ is percolated for all time, then for $d(u,v)$ large,

$$\frac{T(u,v)}{d(u,v)} \sim 0$$

with high probability.

- Information/virus dissemination delay scales
  - linearly with distance if dynamic network is in subcritical.
  - sub-linearly with distance if dynamic network is supercritical.

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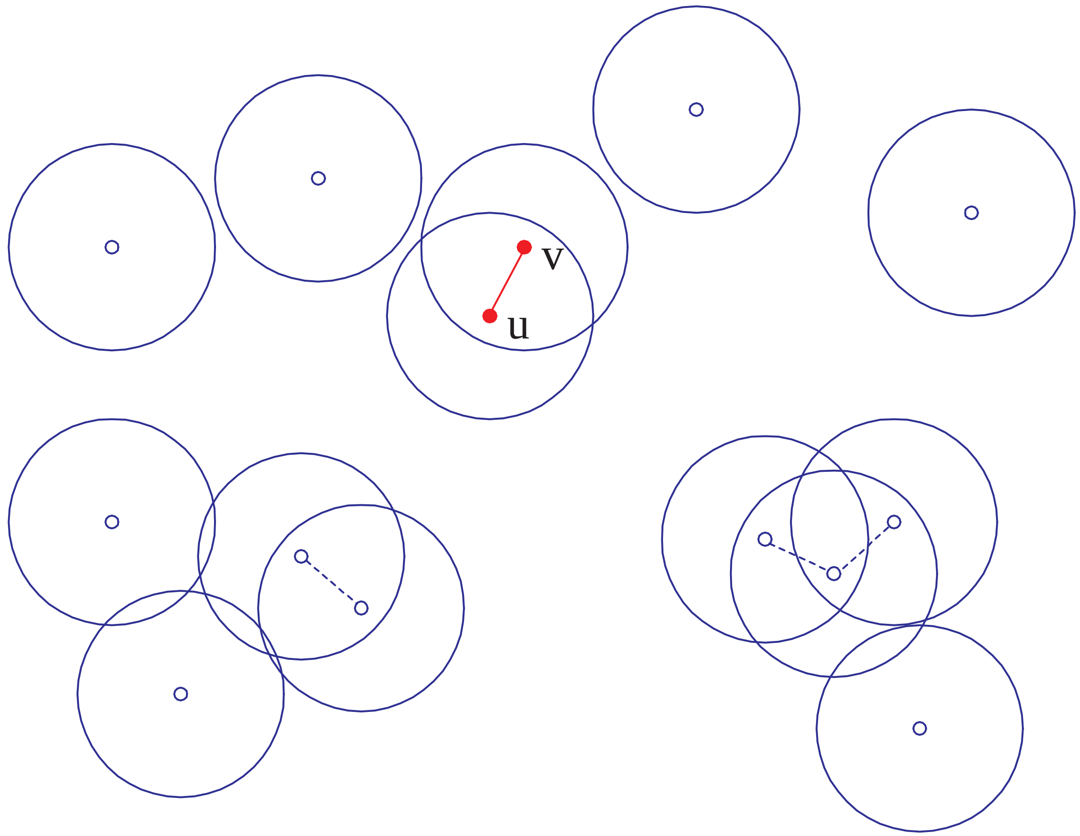
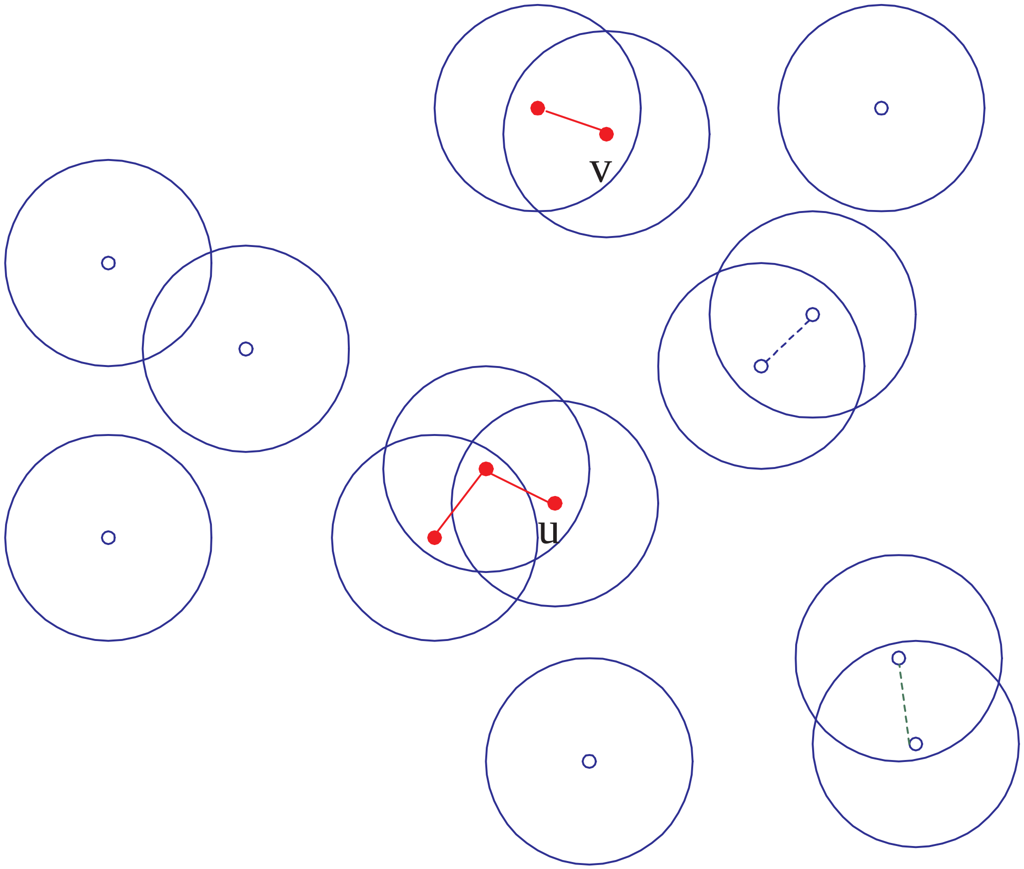
Information/Virus Spread in Mobile Networks



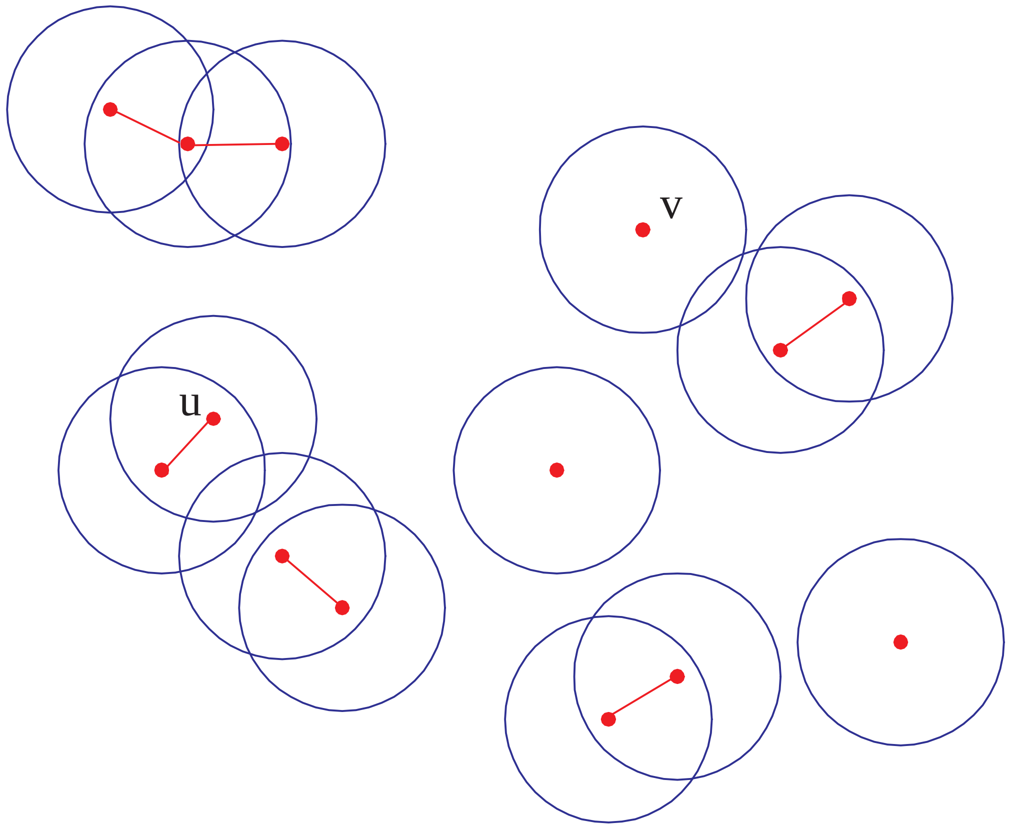
At $t = 0$: network not percolated; source node u broadcasts message.

Information/Virus Spread in Mobile Networks



As t increases, message passed from moving information-carrying nodes to new nodes whenever they are within communication range.

Information/Virus Spread in Mobile Networks



When network is well connected in "mobility" sense, eventually a large fraction of the network, or even the whole network gets message.

Delay Scaling in Mobile Networks

Theorem: Given $G(\mathcal{X}^{(0)})$ under constrained i.i.d. mobility model with $a > 1/2$ and $\lambda > \frac{\lambda_c}{(2a+1)^2}$,

(i) if $\lambda < \lambda_c$, i.e., $G(\mathcal{X}^{(t)})$ is not percolated at any time, then for $d_0(u,v)$ large,

$$\frac{T(u,v)}{d_0(u,v)} \sim \beta$$

with high probability.

(ii) if $\lambda > \lambda_c$, i.e., $G(\mathcal{X}^{(t)})$ is percolated for all time, then for $d_0(u,v)$ large,

$$\frac{T(u,v)}{d_0(u,v)} \sim 0$$

with high probability.

- Information/virus dissemination delay scales
  - linearly with initial distance if mobile network is in subcritical.
  - sub-linearly with initial distance if mobile network is supercritical.

Mobility (Dis)advantage

- Static networks can spread information/virus within giant component of $G(\mathcal{H}_\lambda, 1)$ provided $\lambda > \lambda_c$.
- Mobile networks can spread information/virus within giant component of long-term connectivity graph G', provided $\lambda > \frac{\lambda_c}{(2a+1)^2}$.

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Network Robustness

- In many large-scale networks, nodes and links are vulnerable:
  - Wireless devices infected by viruses/worms.
  - Power networks with unreliable generators and lines.

- Network security severely compromised by cascading failures:
  - Network-wide computer virus/worm epidemic
  - Power blackouts in electricity grid (losses of $100 billion/year in U.S.)

- Essential point: assess whether network has been affected in global manner, rather than in isolated local manner.

- Percolation theory (existence of a giant failed component) is appropriate notion.

- Cascading node failures: sequence of node failures triggered by an initial failure resulting in a giant component of failed nodes in network.

Cascading Failure Model

- Given $G(\mathcal{H}_\lambda, 1)$ with $\lambda > \lambda_c$.
- Network is seeded by an initial failure.
- Each node i has susceptibility threshold ψ_i i.i.d. $\sim f(\psi)$.
- Due to infection spread or redistribution of load, each node i fails if (at least) a fraction ψ_i of its neighbors fail.

Cascading Failure

Theorem: (i) Given $G(\mathcal{H}_\lambda, 1)$ with $\lambda > \lambda_1 > \lambda_c$, there exists $k_0 < \infty$ depending on λ and λ_1, s.t. if

$$F_\psi\left(\frac{1}{k_0}\right) \geq \frac{\lambda_1}{\lambda},$$

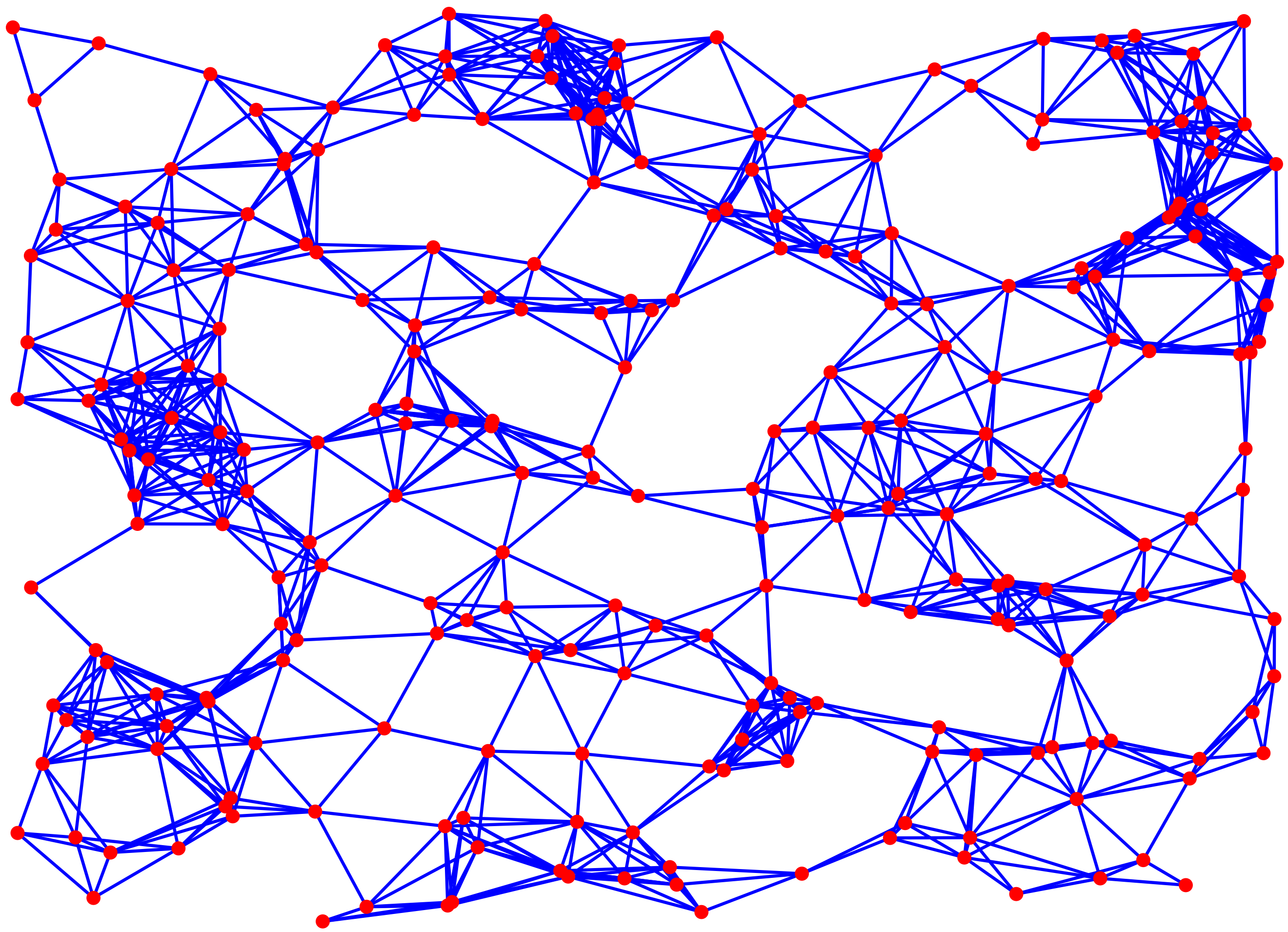
then w.p.1, there is a giant component of vulnerable nodes. If initial failure is inside or adjacent to this component, there exists a cascading failure.

(ii) Given $G(\mathcal{H}_\lambda, 1)$ with $\lambda > \lambda_c$, if

$$\sum_{k=1}^{\infty} \frac{\left(\frac{\lambda}{2}\right)^k}{k!} e^{-\frac{\lambda}{2}} \sum_{m=0}^{\infty} \frac{[\lambda(2\sqrt{2}+\pi)]^m}{m!} e^{-\lambda(2\sqrt{2}+\pi)} \left(1 - \left[1 - F\left(\frac{m+k-2}{m+k-1}\right)\right]^k\right) < \frac{1}{27},$$

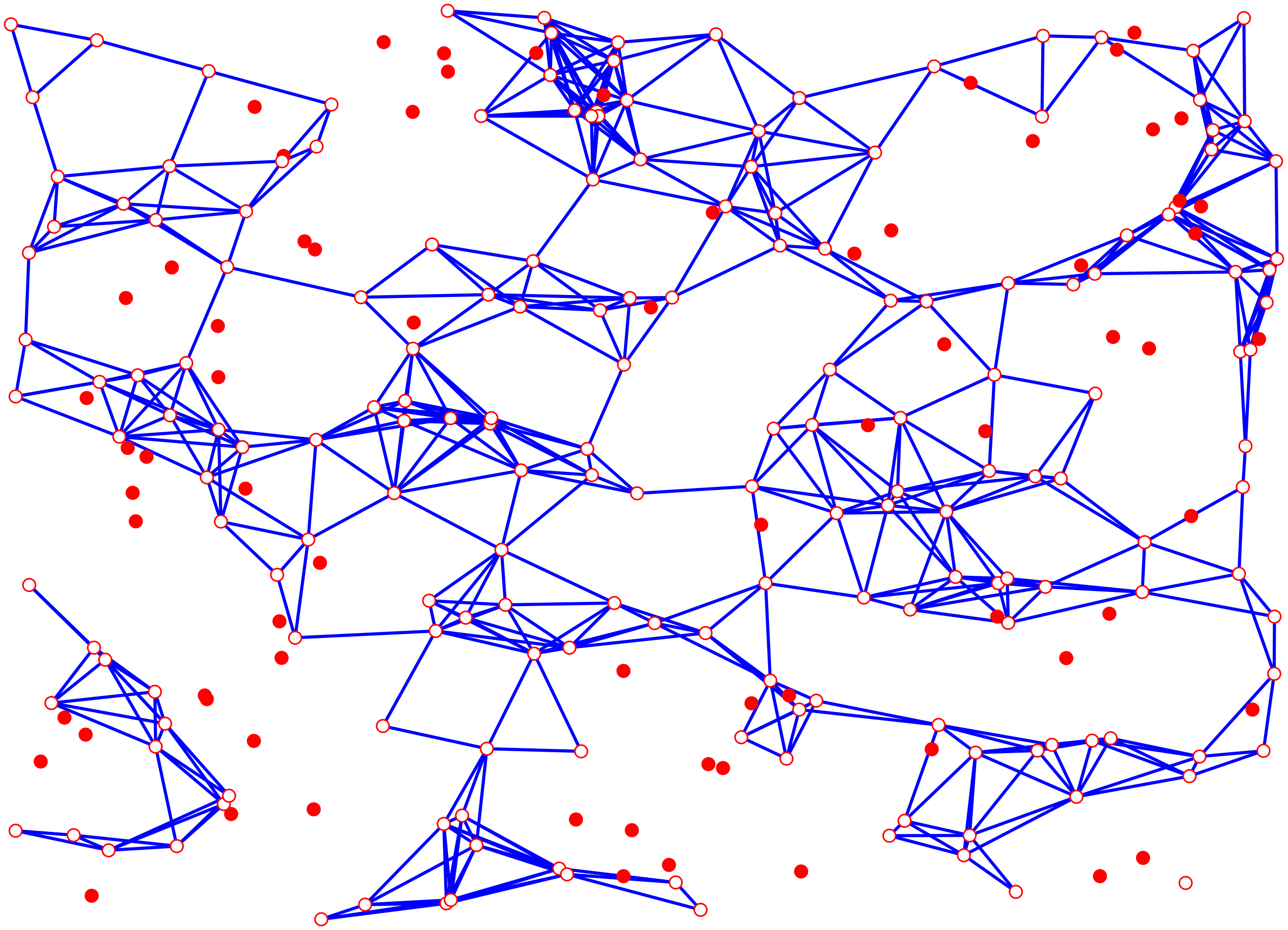
then w.p.1, there is no giant component of unreliable nodes, and no cascading failure occurs no matter where initial failure is.

Example of Cascading Failure



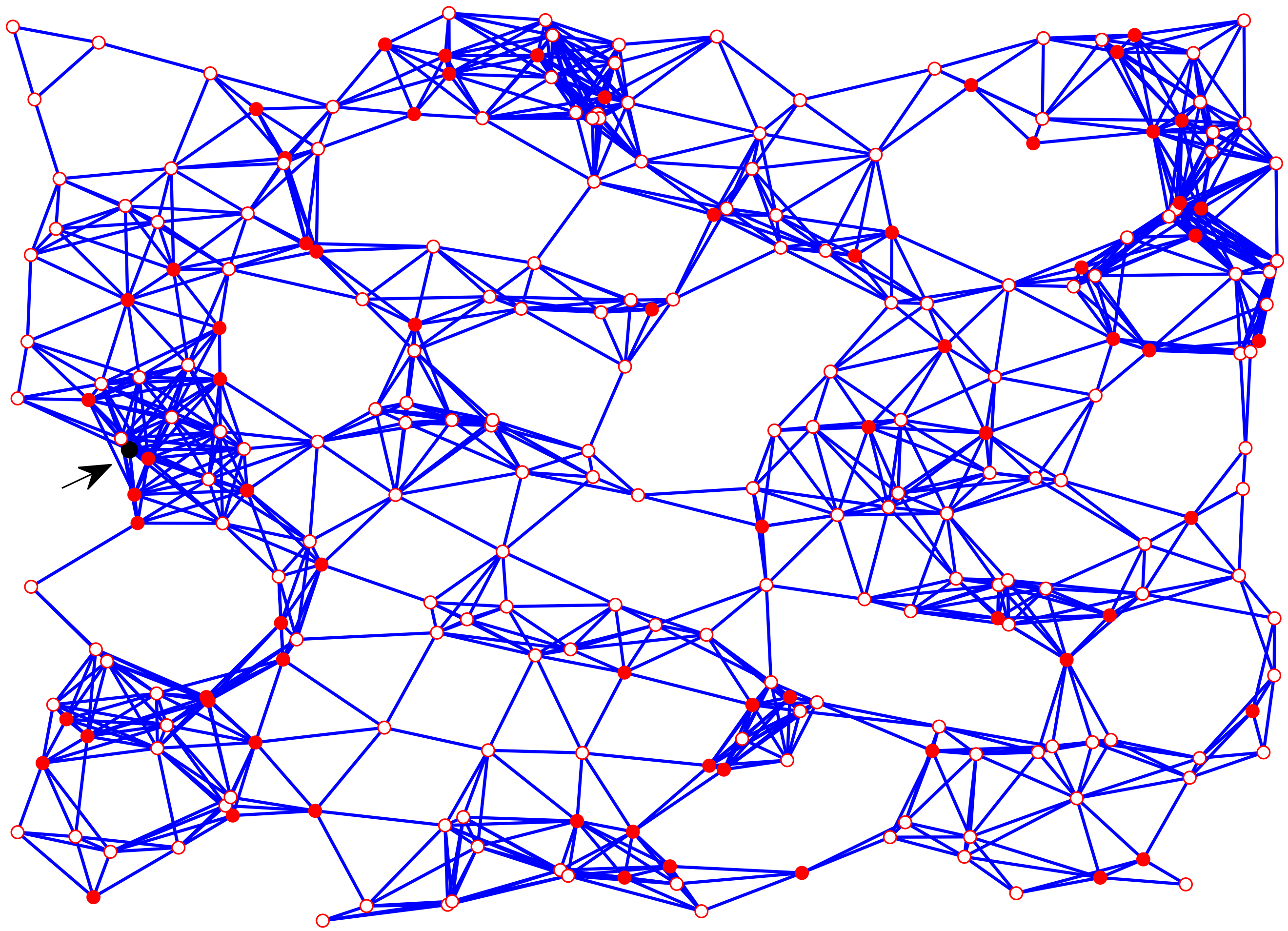
- Initial network: $G(\mathcal{H}_\lambda, 1)$ with $\lambda\pi = 10$.

Example of Cascading Failure



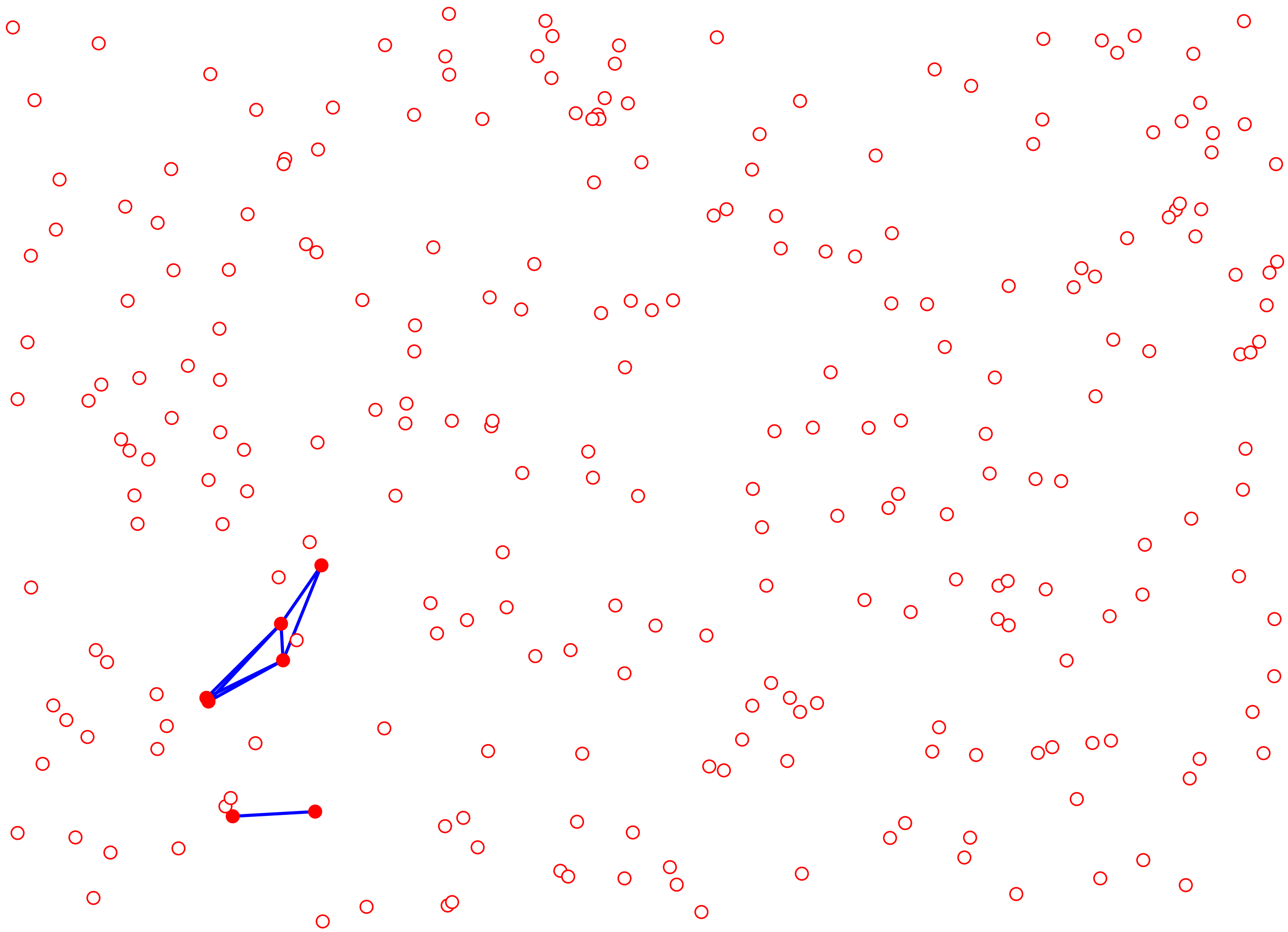
- Component of vulnerable nodes: $f(\psi_i) = \frac{15}{2}$ for $0 < \psi_i \leq 0.1$, and $f(\psi_i) = \frac{5}{18}$ for $0.1 < \psi_i < 1$. (red: non-vulnerable; empty: vulnerable)

Example of Cascading Failure



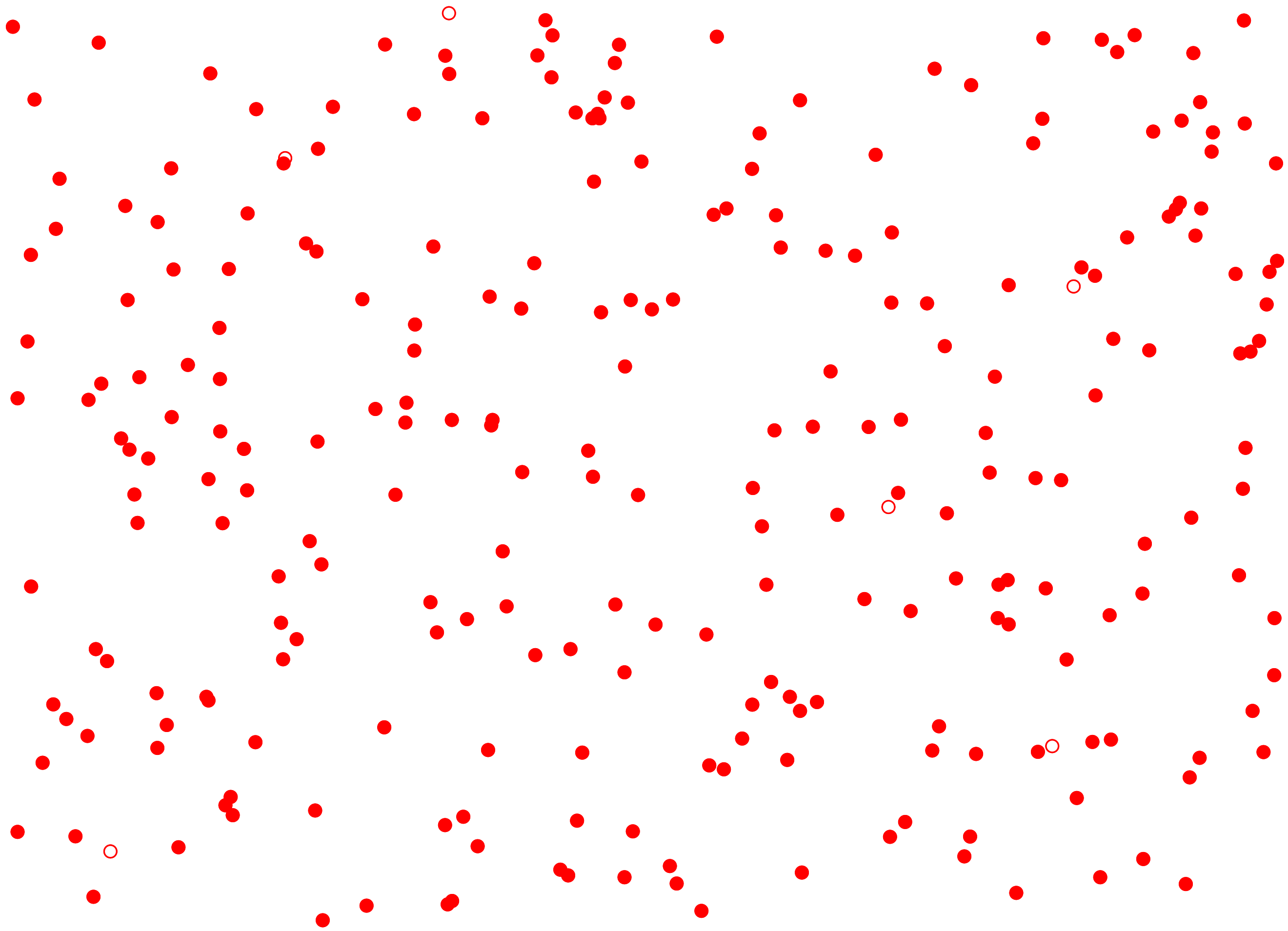
- Initial failure: black node pointed by arrow.

Example of Cascading Failure



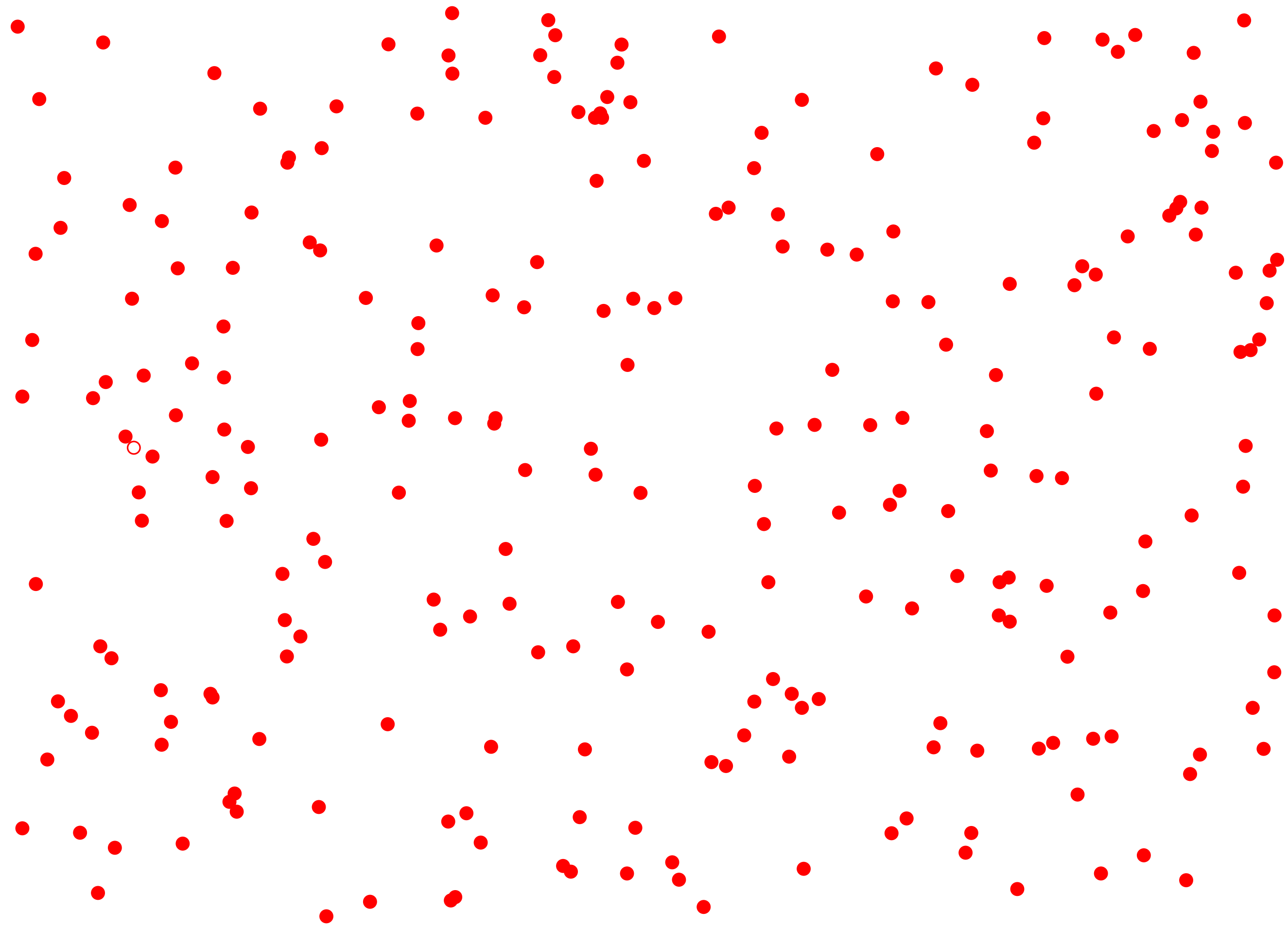
- Cascading failure occurs. (red: operational; empty: failed).

No Cascading Failure



- Same initial network, but $f(\psi_i) = \frac{1}{999}$ for $0 < \psi_i \leq 0.999$, and $f(\psi_i) = 999$ for $0.999 < \psi_i < 1$. (red: reliable; empty: unreliable)

No Cascading Failure

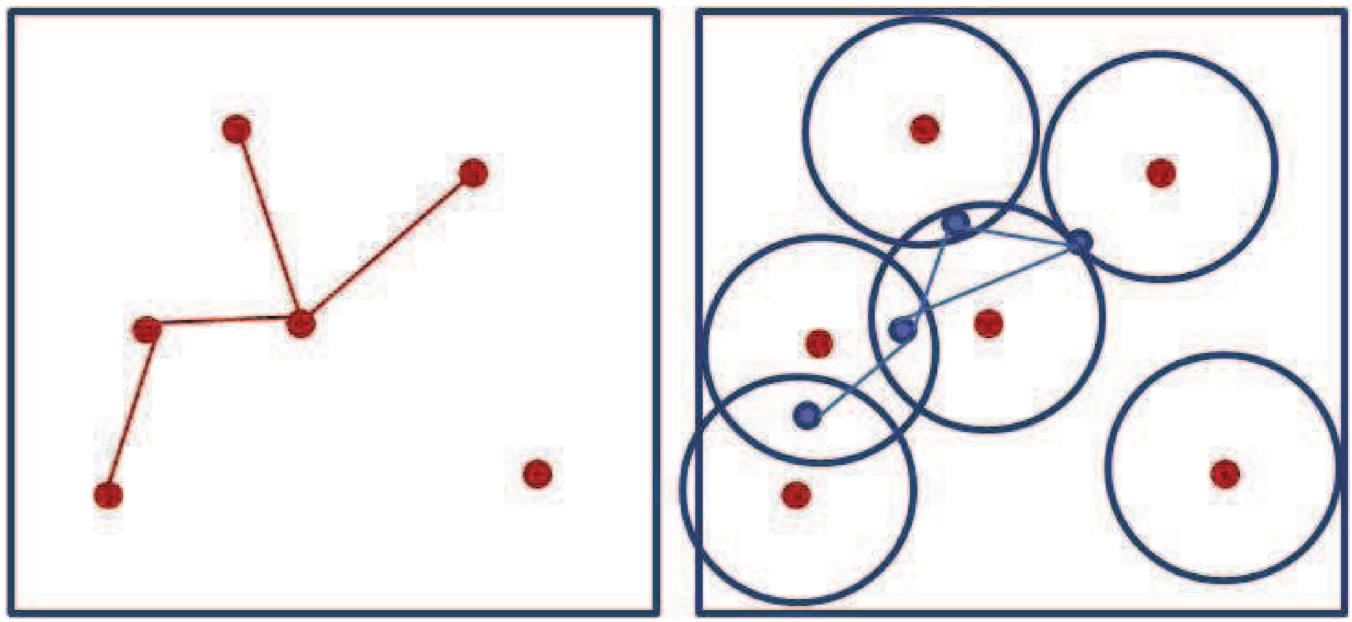


- Same initial failure, no cascading failure occurs. (red: operational; empty: failed)

Link Failures

- Power blackouts often result from line failures.
- Degree-dependent link failure model.
- Degree of link (i, j) = number of links which share an end vertex with $(i, j) = d_i + d_j - 2$.
- Key idea: map links in $G(\mathcal{H}_\lambda, 1)$ to nodes in a covering graph $G_C(\mathcal{H}_\lambda, 1)$.

Covering Graph



- "Boolean model" where coverage radius $= 1/2$.
- Each link in $G(\mathcal{H}_\lambda, 1)$ corresponds to unique node in $G_c(\mathcal{H}_\lambda, 1)$.
- Two nodes in $G_c(\mathcal{H}_\lambda, 1)$ share a link iff corresponding links in $G(\mathcal{H}_\lambda, 1)$ share a common end vertex.

Covering Graph and Link Failures

- Nodes in $G_c(\mathcal{H}_\lambda, 1)$ Poisson-distributed with density $\frac{\pi\lambda^2}{2}$.
- Degree of link in G = degree of corresponding node in G_c.
- $G_c(\mathcal{H}_\lambda, 1)$ not a random geometric graph.
- Key fact: $q_c^{link} \geq q_c^{node}$.
- Can obtain similar results regarding $G(\mathcal{H}_\lambda, 1, q_{link}(k))$ and cascading link failure model.

Conclusions

- In large-scale fixed and mobile wireless computing networks, security and robustness against virus/worms are essential.
- Random geometric graphs and percolation theory provide rigorous mathematical structure.
- Information/virus dissemination delay exhibits linear/sub-linear phase transition for wireless networks with dynamic links.
- Results carry over to mobile wireless networks.
- Characterized conditions for epidemic/cascading failure in geometric networks.
- Common concepts across study of wireless networks and power grids.