Unbiased Estimation of Causal Effects under Network Interference

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Questions

- How can one account for interference between units in an online social network experiment?
- What are the collection of unbiased estimators for causal effects and how can one choose among them?
- How do different aspects of the potential outcomes and the network impact performance?

Potential Outcomes Framework (Neyman-Rubin Causal Model)

- An **allocation** is a vector of $\{0,1\}$-treatments assignments,

$$\mathbf{z} = (z_1, z_2, \ldots, z_n) \in \{0,1\}^n.$$

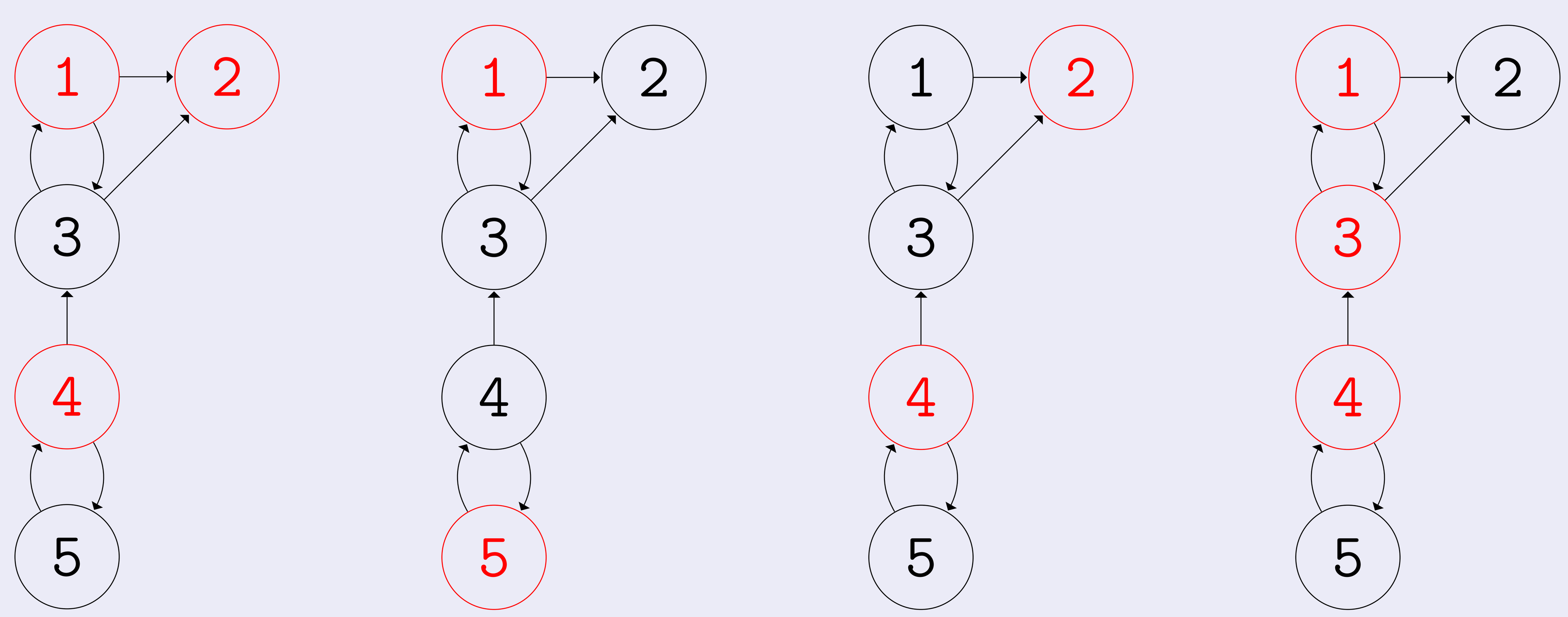
- **Design** is a probability distribution p on $\{0,1\}^n$, with $\mathbf{z}^{obs} \sim p$.
- For allocation $\mathbf{z}$, the **outcome** for unit i is $Y_i(\mathbf{z})$.
  - i.e. a units outcome can depend on all units treatments.
- **Causal estimand** is some function of the potential outcomes.
  - e.g. the average effect of total treatment is $\frac{1}{n}\sum_{i=1}^{n} Y_i(\mathbf{1}) - Y_i(\mathbf{0})$.
  - Core challenge of causal inference is only $Y_1(\mathbf{z}^{obs}), \ldots, Y_n(\mathbf{z}^{obs})$ are observed.

What is network Interference?

No Interference (SUTVA) Y_i depends only on z_i.

Neighborhood Interference Assumption Y_i depends only on z_i and z_j for j in the neighborhood of i.

- Neighborhood is given by a (directed) network $g \in \{0,1\}^{n\times n}$.



$Y_1(\mathbf{z})$ $\quad = Y_1(\mathbf{z}')$ $\quad \neq Y_1(\mathbf{z}'')$ $\quad \neq Y_1(\mathbf{z}''')$

Additional Assumption

Additivity of Main Effects Effect of own treatment and neighbors' treatments are additive.

Additivity of Interference Effects Effects of each neighbor are additive.

Symmetrically Received Interference Outcome only depends on number of treated neighbors.

Symmetrically Sent Interference Effect of treatment of unit j is the same for each unit with j as a neighbor.

An example parameterization: SANIA

If we assume additivity of main effects and symmetrically received interference effects:

$$Y_i(\mathbf{z}) = \alpha_i + \beta_i z_i + \Gamma_i(d_i^{\mathbf{z}})$$

where

- $\alpha_i \in \mathbb{R}$ is the baseline outcome,
- $\beta_i \in \mathbb{R}$ is the "direct treatment effect",
- $\Gamma_i : [n-1] \mapsto \mathbb{R}$ is the interference effect function, and
- $d_i^{\mathbf{z}}$ is the number of treated neighbors of i.

Suppose we want to estimate $\bar{\beta} = \frac{1}{n}\sum_i \beta_i$.

When do unbiased estimate of $\bar{\beta}$ exist?

We consider linear estimates of the form

$$\hat{\beta}^{\mathbf{w}} = \sum_i w_i(\mathbf{z}^{obs}) Y_i(\mathbf{z}^{obs}).$$

Proposition: *LUEs exist under SANIA if and only if for each $i \in [n]$ there exist allocations $\mathbf{z}, \mathbf{z}'$ such that $p(\mathbf{z}) > 0$, $p(\mathbf{z}') > 0$, $d_i^{\mathbf{z}} = d_i^{\mathbf{z}'}$, $z_i = 1$, and $z_i' = 0$.*

Minimum Integrated Variance LUEs

- We propose minimum integrated variance (MIV) LUE for some "prior" distribution over the parameters α_i, β_i, Γ_i.
- For priors with independence across units, the MIV LUE is Horvitz-Thompson like, with inverse propensity score weighting.
- For priors where parameters are equal across units, the MIV LUE is similar to a naive estimate, where one takes the difference of means between the treatment and control groups, stratified on the number of treated neighbors.

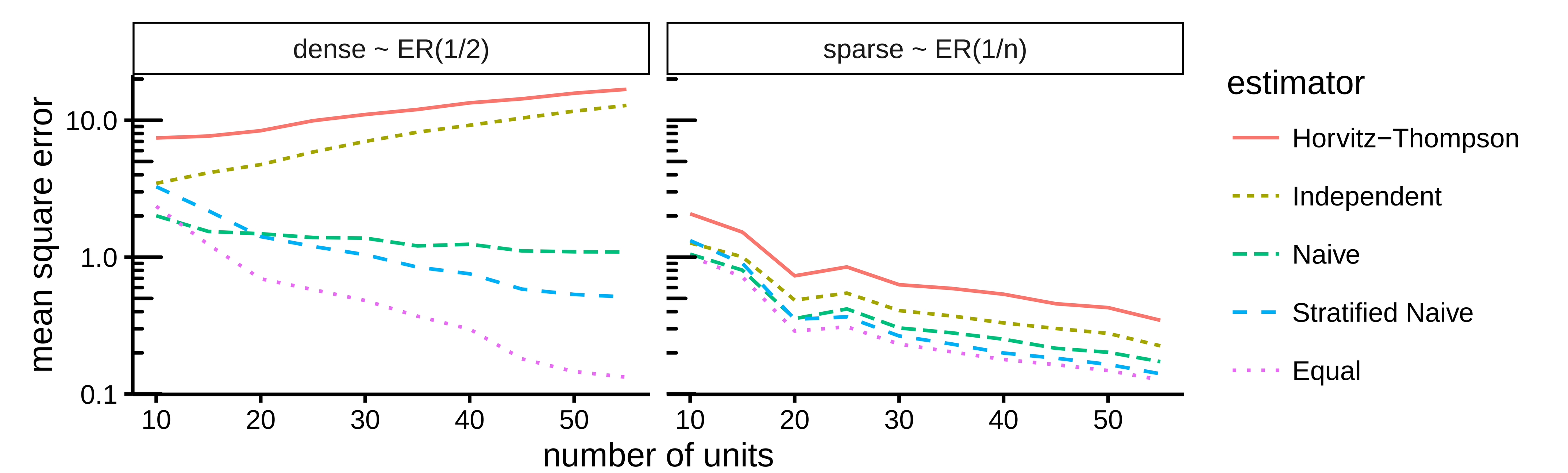


Figure: Mean square errors on the log scale versus the number of units, for dense graphs and sparse graphs. Independent is a MIV LUE where all parameters are independent. Equal is a MIV LUE where parameters are equal across units. Naive and Horvitz-Thompson are the two standard estimators under no interference and Stratified Naive an average of naive estimators conditioned on the number of treated neighbors.

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