Causal Inference with Partially Revealed Interference

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Overview

The interpretation of experiments is complicated when the outcome of an experimental unit depends not only on its assigned treatment but also on interferences from other units.

In our setting, this interference can depend on units' characteristics and the treatment assignment itself, and is often only **partially revealed.**

To assess causal effects we **model** interference between units **as a network**, and develop novel testing procedures that involve repeated sampling of the treatment assignment under constraints from the network topology and the tested hypothesis.

We illustrate our causal framework in applications where such forms of interference are ubiquitous but currently not adequately addressed.

Background

Consider N units, $Z \in \{0,1\}^N$ treatment assignment, $G \in \{0,1\}^{N \times N}$ social network between units, $X_i \in \mathbb{R}^p$ covariates of unit i, and $Y_i(Z)$ outcome of unit i under assignment Z.

No interference. [Rubin, 1974]

$$Y_i(Z) = Y_i(Z') \text{ if } Z_i = Z_i'.$$

No foreign interference. [Manski, 1993, Toulis and Kao, 2013]

$$Y_i(Z) = Y_i(Z') \text{ if } Z_i = Z_i' \text{ and } Z_{G_i} = Z'_{G_i}.$$

Classical assumptions:

- The no interference assumption is untenable when a treatment is applied on networked units.
- The assumption of no foreign interference can be **overly restrictive**. For instance, a unit i might still interfere with a foreign unit j if, say, they have similar interests.
- Generally, the *source* of interference can depend on units' characteristics, X and G, or the treatment assignment Z—this situation is incompatible with no foreign interference assumptions.

Our framework & notation

Let $s_{ij}(Z) = 1$, if unit j **interferes** with unit i under Z, and $s_{ij}(Z) = 0$ otherwise; Let $S_i(Z) = (s_{ij}(Z) : j = 1, \ldots, N)$, and $S(Z) = (s_{ij}(Z) : i, j)$.

Let $\mathsf{s}_{ij} = 1$ if $s_{ij}(Z) = 1$ for some Z, i.e., *j can interfere* with i; let $\mathsf{s}_i = (\mathsf{s}_{ij} : j)$ and $\mathsf{s} = (\mathsf{s}_{ij} : i, j)$ denote *possible* interference for unit i and all units, respectively.

Under assignment Z, every unit i is **exposed** to composite treatment $D(Z_i, S_i(Z)) \equiv D_{iZ} \in \mathcal{D}$, where $\mathcal{D}$ is a set of **treatment exposure levels**.

Example. Suppose the treatment is a movie ad on movie fans (units) who can message each other. The exposure levels could be $\mathcal{D}$ = {"AD+MSG", "AD + NO MSG", "NO AD + MSG", "NO AD + NO MSG"}, which indicate, accordingly, whether a unit watched or not the ad, and whether the unit received or not a message from another unit. In this case,

$$D_{iZ} = \begin{cases} \text{"AD + MSG"} & \text{if } Z_i = 1 \text{ and } \sum_{j\in} s_{ij}(Z) > 0, \\ \text{"AD + NO MSG"} & \text{if } Z_i = 1 \text{ and } \sum_{j\in} s_{ij}(Z) = 0, \\ \text{"NO AD + MSG"} & \text{if } Z_i = 0 \text{ and } \sum_{j\in} s_{ij}(Z) > 0, \\ \text{"NO AD + NO MSG"} & \text{if } Z_i = 0 \text{ and } \sum_{j\in} s_{ij}(Z) = 0. \end{cases}$$

Method

Assumption. [Stable Treatment Exposure] *The potential outcome $Y_i(Z)$ under assignment Z satisfies*

$$Y_i(Z) = Y_i(Z') \textit{ if } D_{iZ} = D_{iZ'}.$$

This assumption (SUTEVA) generalizes previous approaches:

- If $D_{iZ} = Z_i$ then SUTEVA reduces to classical SUTVA [Cox, 1958, Rubin, 1980].
- If $D_{iZ} = D_{iZ'}$ when $Z_{G_i} = Z'_{G_i}$—i.e., interference to i is fixed if friends' treatment is fixed—then SUTEVA reduces to the no foreign interference assumption.

Definition. [Causal Estimand]

$$\tau_{ab} = \text{Ave}\left(Y_i(Z) - Y_i(Z') | D_{iZ} = a, D_{iZ'} = b\right)$$

Note. The causal estimand τ_{ab} is well-defined only on units who *can* be exposed to both levels a and b, i.e, on the set $U_{ab} = \{i : \exists Z, Z' \text{ s.t. } D_{iZ} = a \text{ and } D_{iZ'} = b\}$.

Hypothesis H_0: For every Z, Z' and every unit in U_{ab},

$$Y_i(Z) = Y_i(Z') \text{ if } \{D_{iZ}, D_{iZ'}\} \subseteq \{a, b\}$$

Testing procedure.

1. Impute the missing connections s^{mis} conditional on $\mathsf{s}^{\text{obs}}, X, G$, and Z—define $\mathsf{s}^{\text{imp}} = \mathsf{s}^{\text{obs}} + \mathsf{s}^{\text{mis}}$ as the complete possible connections.
2. Calculate test statistic T^{obs} on $Y^{\text{obs}}, \mathsf{s}^{\text{obs}}, Z, X, G$.
3. Sample a new treatment assignment Z'.
4. Under H_0, impute counterfactual $Y_i(Z')$ for all units in the set U_{ab}.
5. Apply an *expression model* Q to get observed interference under Z',
$$S_i(Z') = Q(\mathsf{s}_i^{\text{imp}}, Z').$$
6. Discard s^{imp} and repeat from Step 1 with Z' as the treatment assignment, and $S_i(Z')$ and $Y_i(Z')$ as observed data.

Application

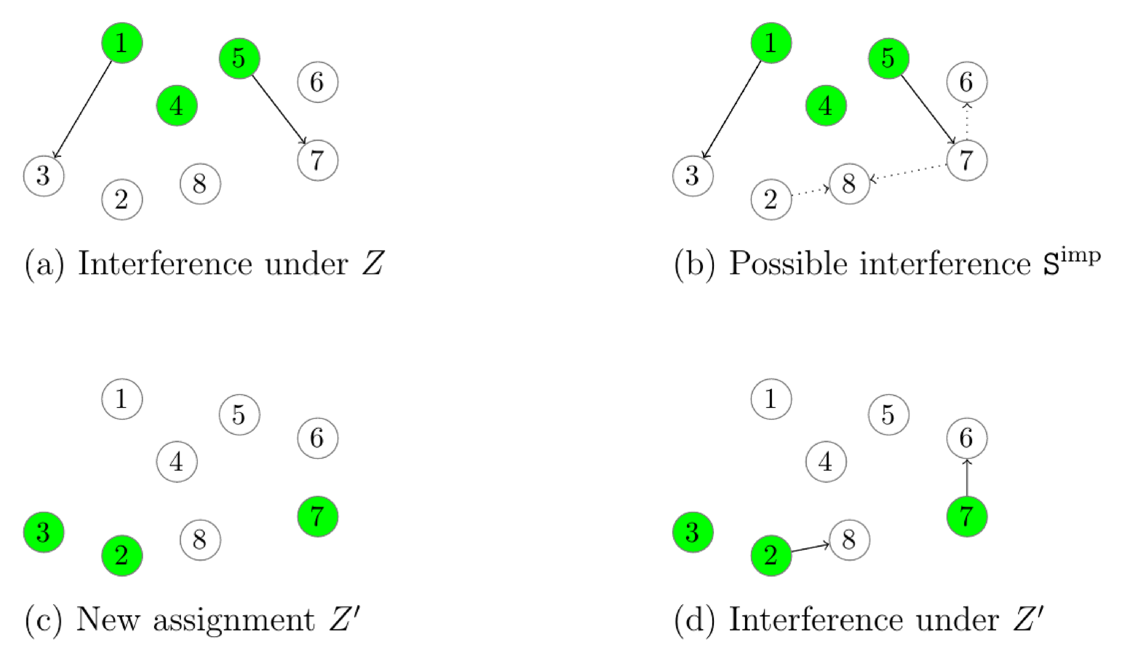
Kim et al. [2015] applied a treatment on a village, which determined how 5% of the villagers (the *seeds*) was selected to receive training on proper health practices. There were three treatments, namely, (1) selecting the seeds randomly, (2) selecting most social seeds, and (3) selecting seeds based on nomination. Seeds received training on health practices, which were propagated by sharing coupons for discounted health products—interestingly, there was **minimal overlap** between coupon passing and the social network.

- **Units:** Individuals in a village.
- **Z:** Seed ($Z_i = 1$) or not ($Z_i = 0$).
- **G**: Social network in village.
- **X**: Personal info (age, education)
- **Interference:** $s_{ij}(Z) = 1$ if j gives coupon to i under assignment Z.
- **Exposure levels**: $\mathcal{D}$ = {"seed", "no seed + coupon", "no seed + no coupon"}.

We test (for non-seeds) whether

$$Y_i(\text{"no coupon"}) = Y_i(\text{"coupon"})$$

Illustration of testing procedure



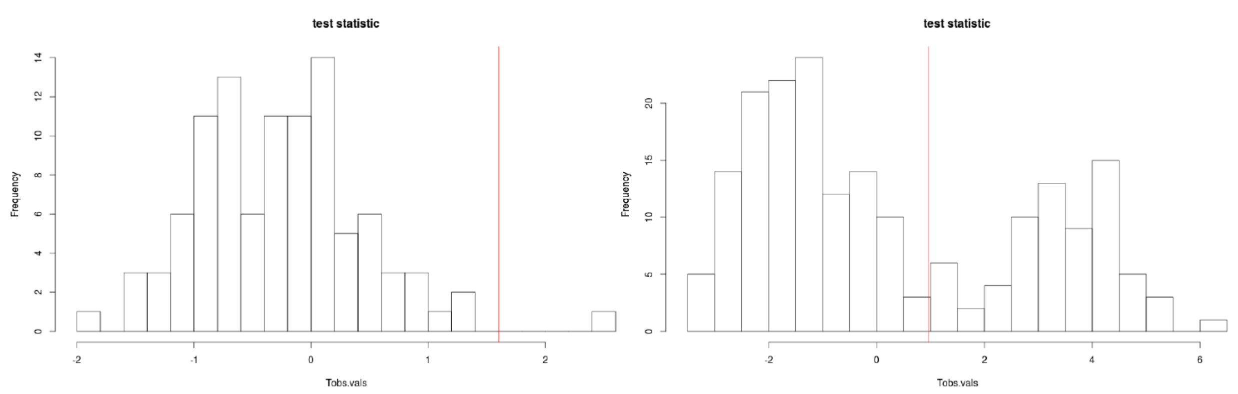
(a) Interference under Z

(b) Possible interference s^{imp}

(c) New assignment Z'

(d) Interference under Z'

Results (left: social seeds, right: random seeds)



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