# Secure Multi-Party Computation of Probabilistic Threat Propagation

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# Problem: Collaborative Cyber Defense

**Companies should share cyber threat information with each other and USG to prevent attacks**

**Sector-Specific Sharing Centers**

FINANCIAL SERVICES Information Sharing and Analysis Center

DIB ISAC
DEFENSE INDUSTRIAL BASE
INFORMATION SHARING AND ANALYSIS CENTER

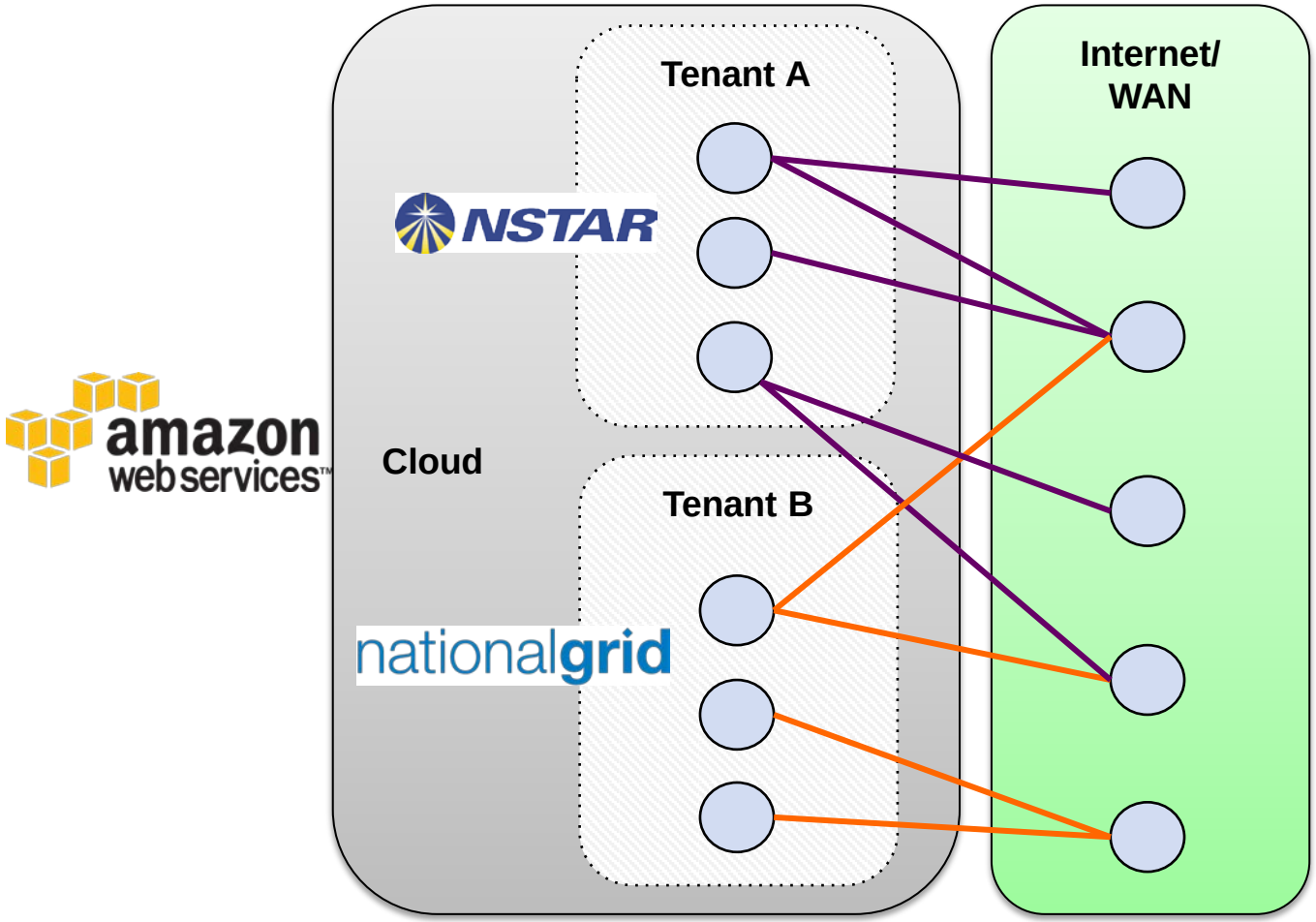
AVIATION ISAC

HEALTH ISAC

- **Global cyber situational awareness benefits all**
- **However, cybersecurity information is sensitive**
  - **Prior attacks may indicate weaknesses in defense**
  - **Business information could be used by competitors**

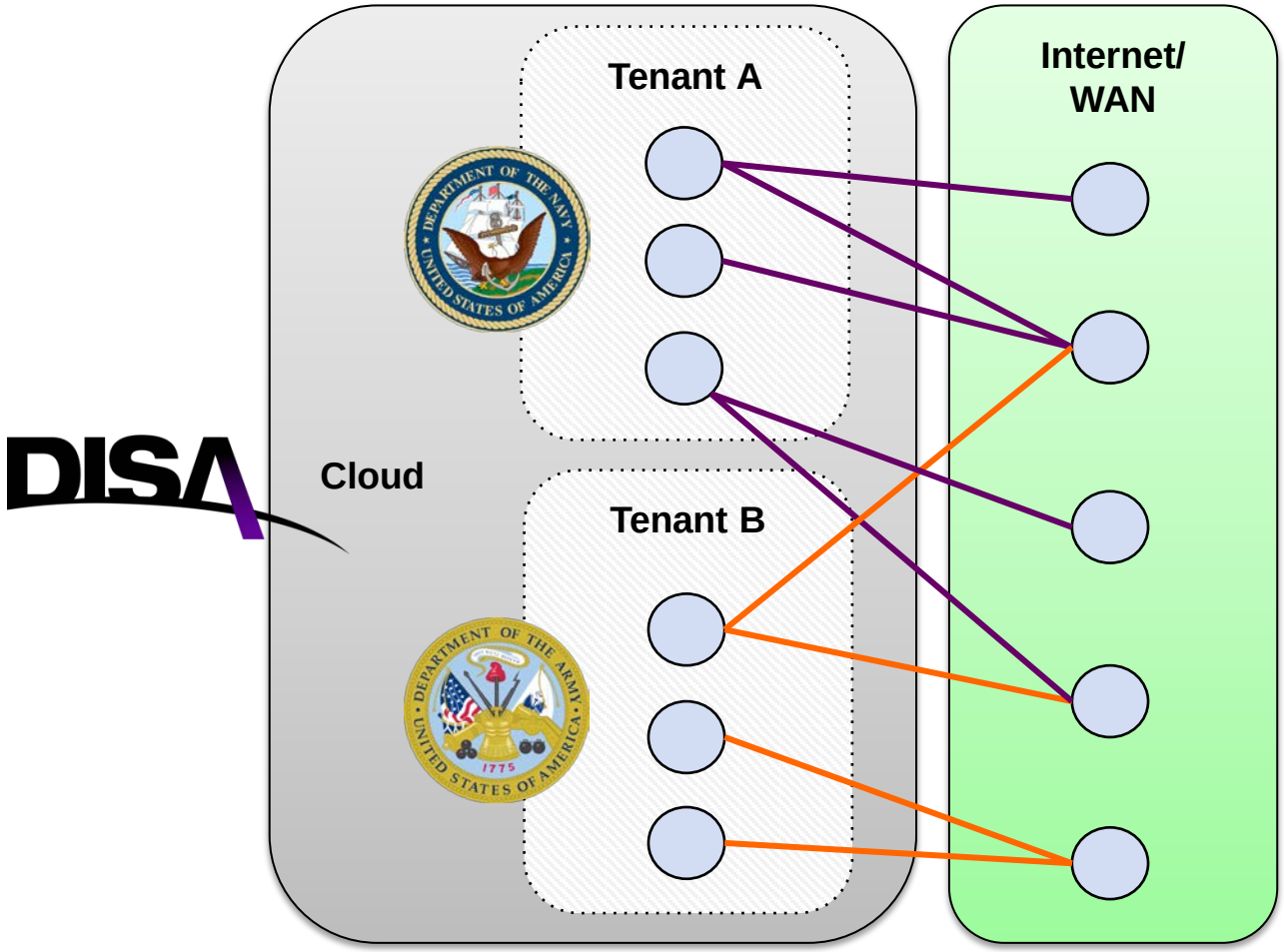
**Goal: Compute joint analytics without sharing sensitive cyber security information**

# Cloud Threat Sharing



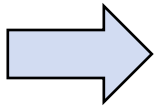

**Cloud introduces provider and multiple tenants with varying trust**

# Cloud Threat Sharing



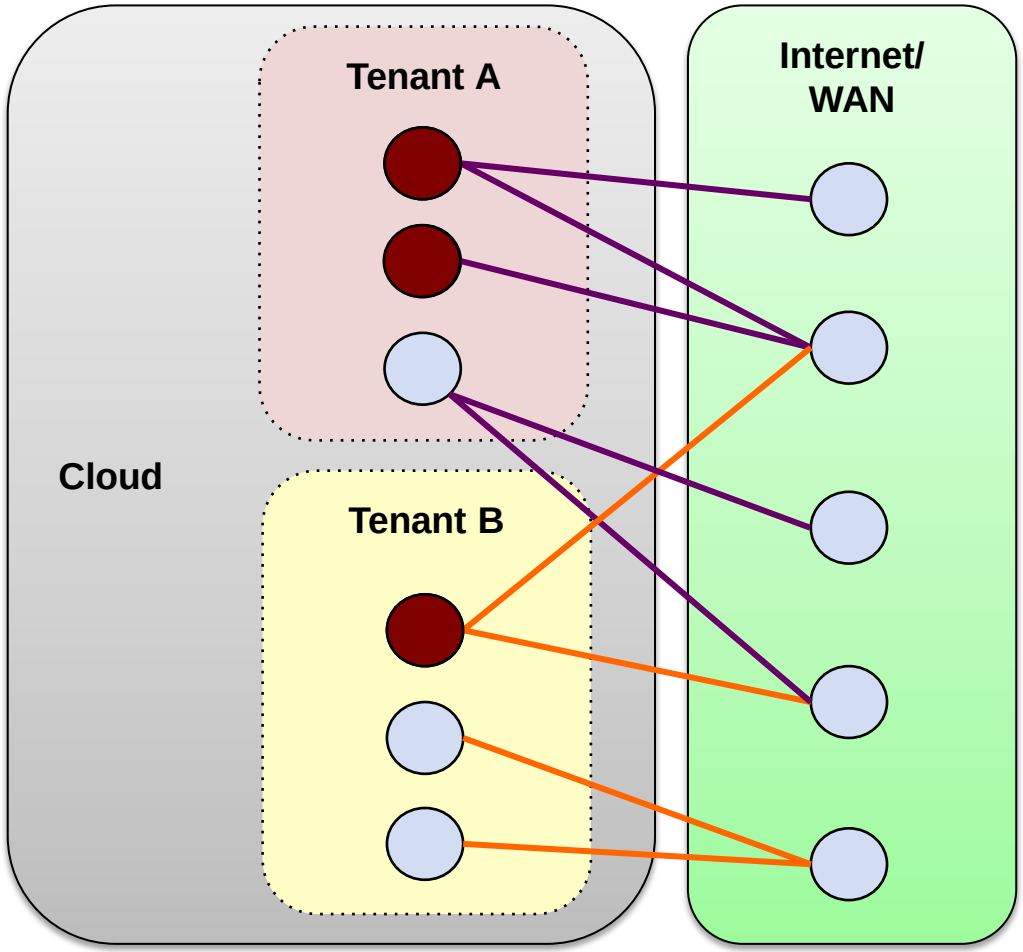

**Cloud introduces provider and multiple tenants with varying trust**

# Outline



- **Motivation**
- **Probabilistic Threat Propagation (PTP)**
- **Secure Multi-Party Computation (MPC)**
- **Application of MPC to PTP**

# Probabilistic Threat Propagation (PTP)



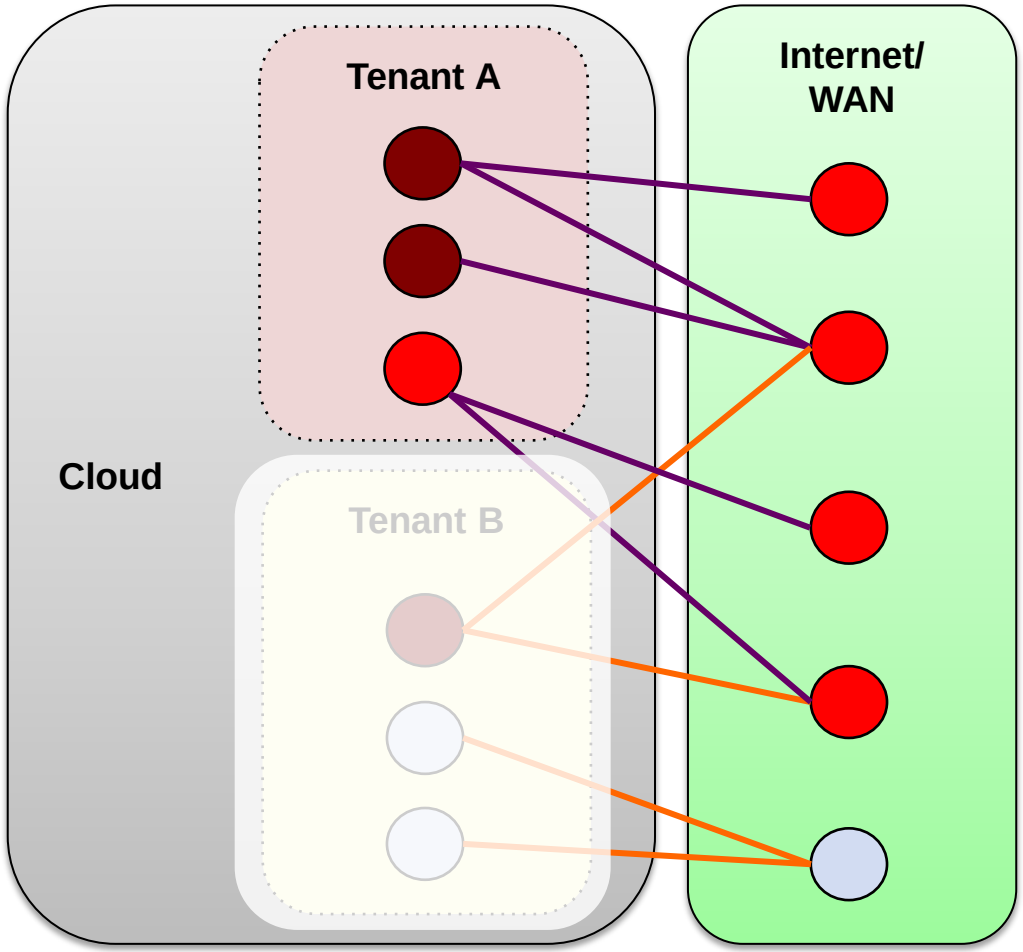

**Require:** $W$, $\{tips\}$, $\gamma$, $\alpha$
$P \leftarrow \alpha^N$, $P(\{tips\}) \leftarrow \gamma$
$C \leftarrow 0^{N \times N}$
**repeat**
$T \leftarrow W \otimes P^T$
$C \leftarrow T - W \circ C^T$
$P \leftarrow < C, \bar{1} >$
$C(\{tips\}, \cdot) \leftarrow 0$
$P(\{tips\}) \leftarrow \gamma$
**until** $P$ has converged
**return** $P$

**Goal: Use connectivity graph and known bad hosts to predict other compromised hosts**



Kevin M. Carter, Nwokedi C. Idika, William W. Streilein: Probabilistic Threat Propagation for Network Security. IEEE Transactions on Information Forensics and Security 9(9): 1394-1405 (2014)

# Probabilistic Threat Propagation (PTP)

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**Tenants don't want to share compromised host info with each other or with provider**



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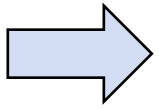

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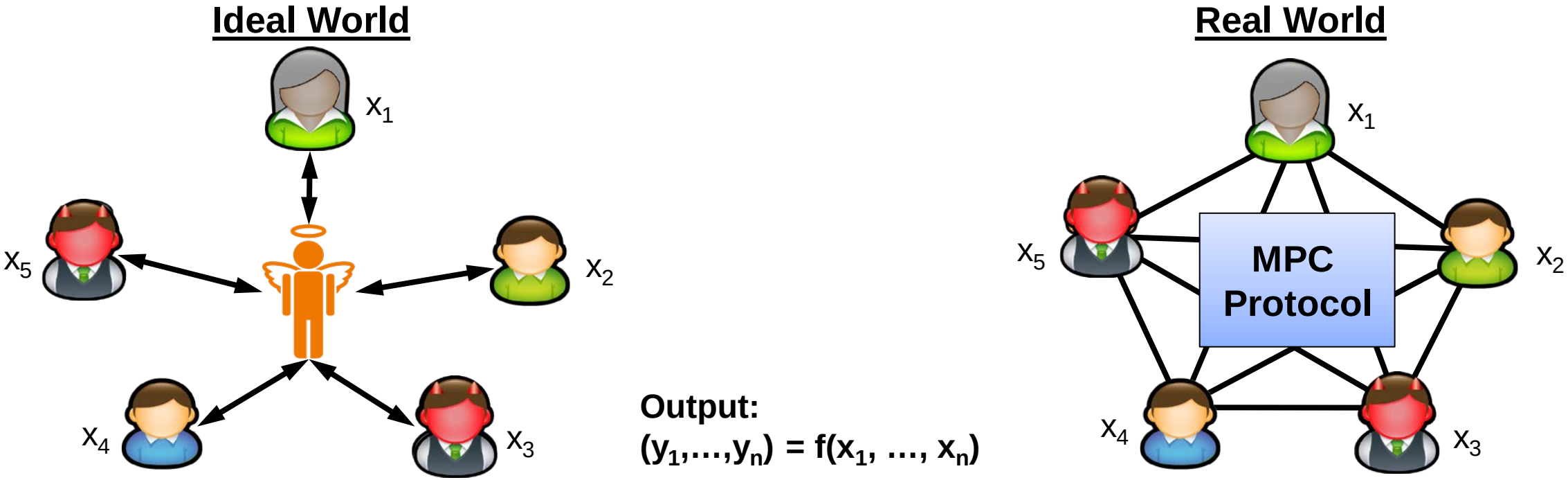
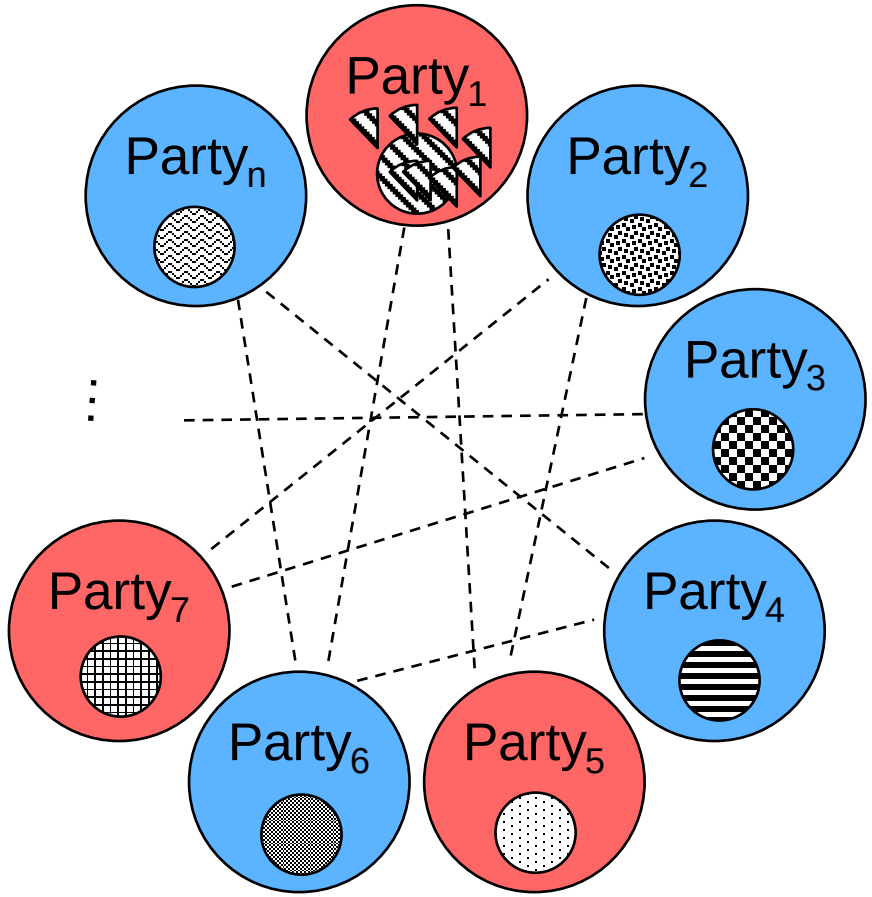
# Secure Multi-Party Computation (MPC)




- **MPC emulates a trusted party, assuming no more than a threshold number of adversarial parties, guaranteeing:**
  - **Correctness of computation**
  - **Confidentiality of inputs and outputs**

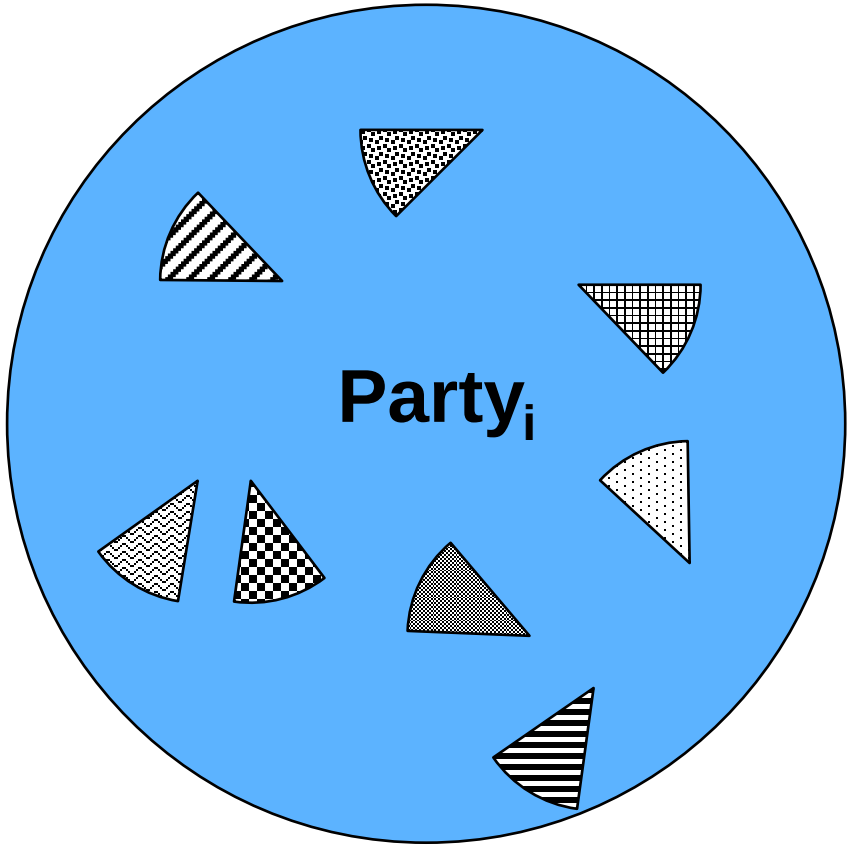
# Secret Sharing Overview




- **Each secret value can be “split” into pieces (“shares”)**
- **If enough people combine shares, they can reconstruct the secret**
- **Otherwise, no one learns any info**

Slide credit: Sasha Berkoff

# MPC Based on Secret Sharing




**To perform MPC, each party:**

- **Sends shares of its secret to each other party**
- **Computes on shares**
  - **Addition: local computation**
  - **Multiplication: requires interaction**
- **Interacts to combine manipulated shares to obtain final answer**

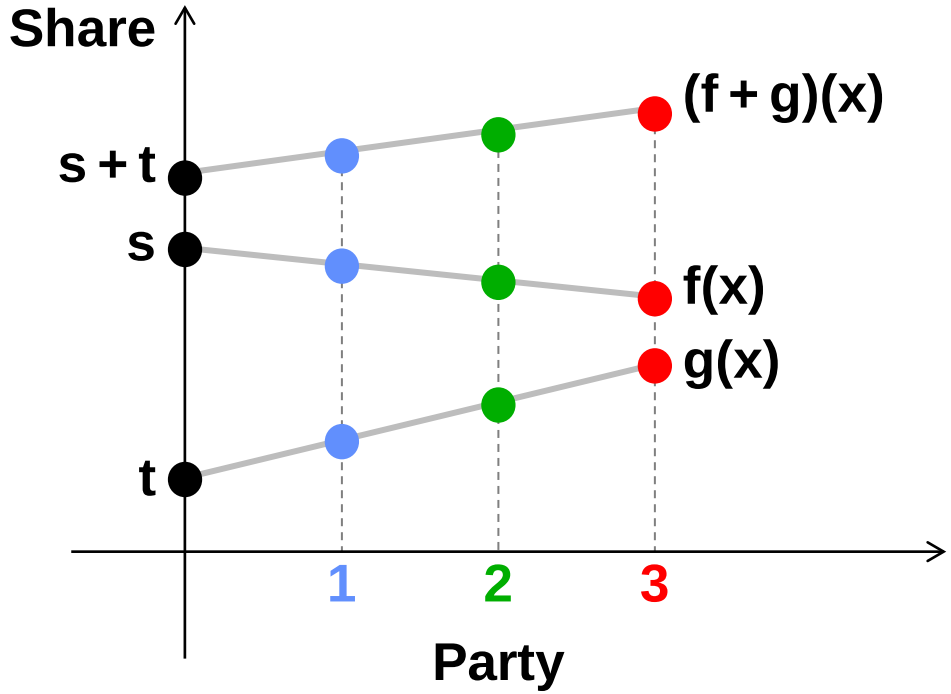
Slide credit: Sasha Berkoff

# Secret Sharing Example

**To secret share for a threshold k:**

- **Give each party a point on a random degree-k polynomial**
- **Reconstruction: interpolate any >k points to recover degree-k polynomial**
- **Addition (s + t): f(i) + g(i)**
- **Multiplication (s × t): f(i) × g(i), then interactive degree reduction**

**Example: threshold k = 1**

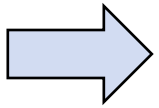



**Secret sharing can be used to compute any arithmetic function securely**



BGW – Ben-or, Goldwasser, Wigderson: "Completeness Theorems for Non-Cryptographic Fault-Tolerant Distributed Computation (Extended Abstract)." STOC 1988

# Outline



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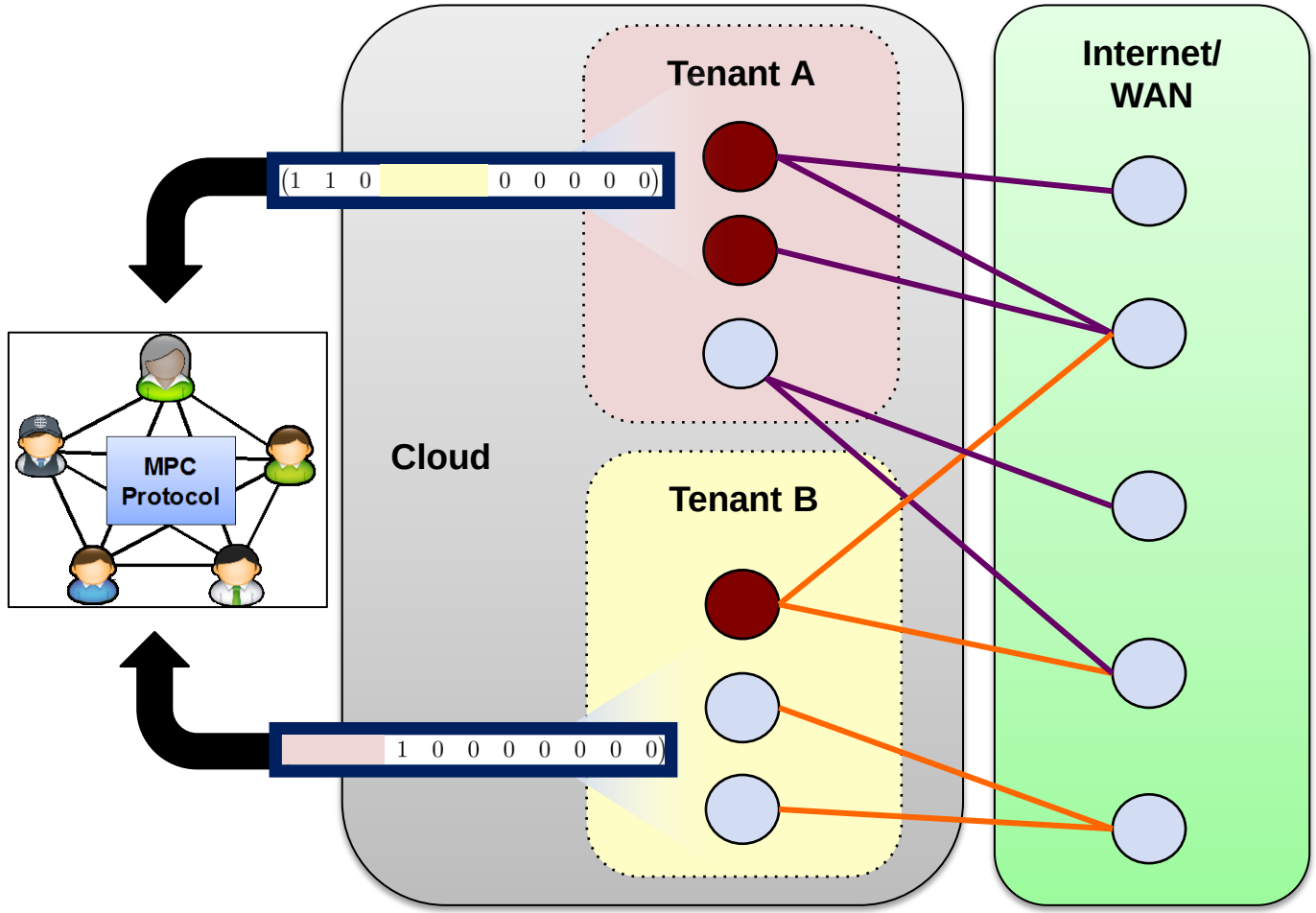
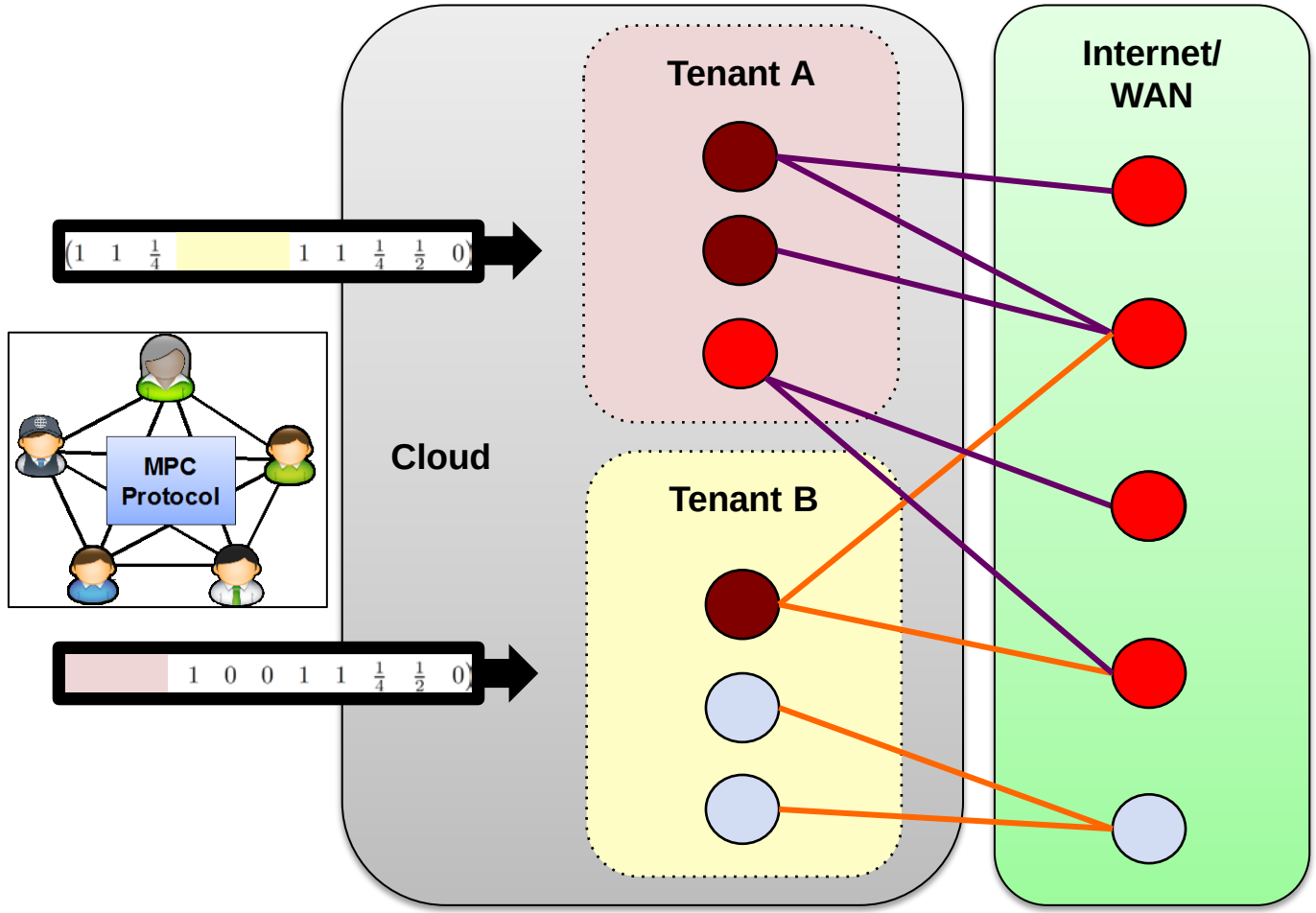
# Secure Multi-Party Computation of PTP




**Tenants secret-share compromised host info with other tenants, jointly compute PTP securely**

# Secure Multi-Party Computation of PTP




**Each party learns about compromised hosts in its own network and on the Internet**

# Secure Multi-Party Computation of PTP

## Probabilistic Threat Propagation Algorithm

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MPC Operations

- $N^2$ mult-by-const
- $N^2$ mult-by-const, adds
- $N^2$ adds
- $N^2$ mults
- N mults, N adds
- N mults, 2N adds, 1 comp

- **Map high-level algorithm to mathematical operations**
- **Low overhead: add; multiply by constants**
- **High overhead: multiply, compare**

# MPC Optimization Techniques

## 1 Fixed Point Arithmetic

**Efficiently compute on non-integer values using secure fixed point arithmetic**

| Representation | $a \to \tilde{a} := \lfloor a \cdot 2^{30} \rfloor$ |
|---|---|
| Addition | $a+b \to \tilde{a}+\tilde{b} \approx \lfloor (a+b)\cdot 2^{30} \rfloor$ |
| Multiplication | $a \cdot b \to \tilde{a}\cdot\tilde{b}/2^{30} \approx \lfloor (a\cdot 2^{30})\cdot(b\cdot 2^{30})/2^{30} \rfloor = \lfloor (a\cdot b)\cdot 2^{30} \rfloor$ |

**All MPC computation done on integer values**

## 2 Oblivious Selection

**Select/assign values to/from secret indices using secret-shared indicator vector**

$$\begin{pmatrix} 1/3 \\ 0 \\ 2/3 \end{pmatrix} * \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2/3 \end{pmatrix}$$

$$\underset{\text{Original shares}}{\begin{pmatrix} 92134 \\ 41431 \\ 19384 \end{pmatrix}} * \underset{\text{Indicator vector}}{\begin{pmatrix} 14321 \\ 3413 \\ 7324 \end{pmatrix}} = \underset{\text{New shares}}{\begin{pmatrix} 34549 \\ 3412 \\ 23412 \end{pmatrix}}$$

**MPC hides selection/assignment indices**

## 3 Sparse Matrices

**Reduce computation on sparse matrices by storing only nonzero entries**

**Original**

$$\begin{pmatrix} 0 & 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 & 6 \\ 2 & 0 & 0 & 2 & 2 \\ 3 & 0 & 3 & 0 & 0 \\ 0 & 3 & 3 & 0 & 0 \end{pmatrix}$$

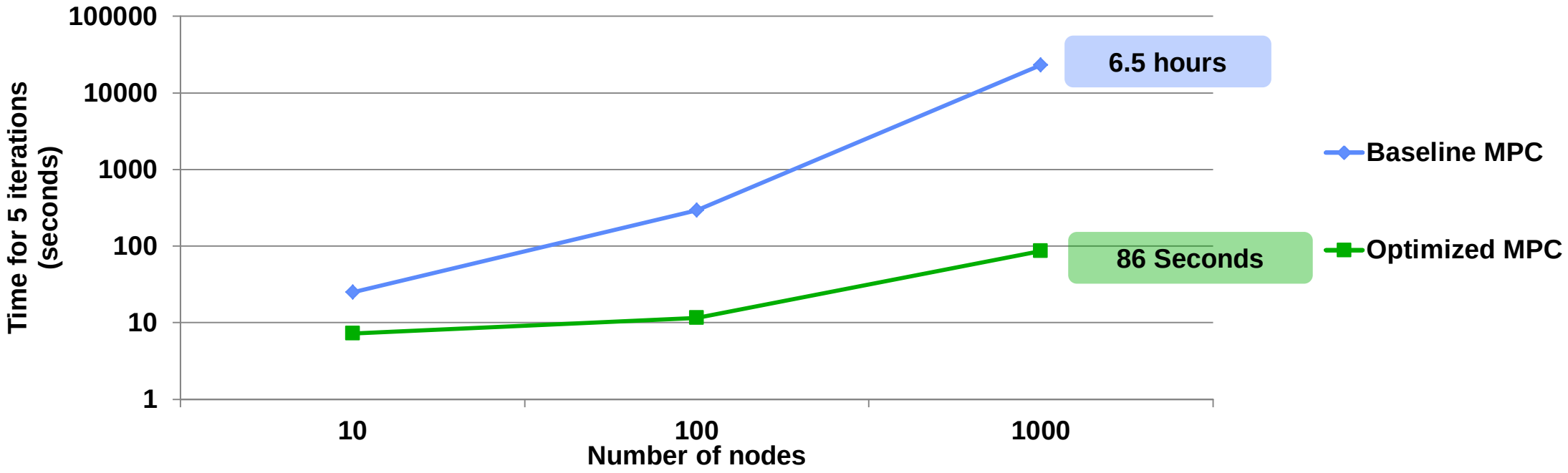
**Sparse**

$$\begin{pmatrix} 3 & 3 & 6 & 2 & 2 & 2 & 3 & 3 & 3 & 3 \\ 1 & 1 & 2 & 3 & 3 & 3 & 4 & 4 & 5 & 5 \\ 3 & 4 & 5 & 1 & 4 & 5 & 1 & 3 & 2 & 3 \end{pmatrix}$$

**Nonzeros**
**Row indices**
**Col indices**

**MPC operations only needed on nonzero entries**

# Performance of Secure Multi-Party Computation of PTP




**Optimizations result in >250× speedup over baseline MPC**

# Summary

- Graph analytics (e.g., probabilistic threat propagation) have important applications to cyber security
- Privacy concerns may restrict ability to perform joint analytics
- Secure multi-party computation enables privacy-preserving computation of analytics
- Designed and optimized MPC for PTP, achieving significant speedup over baseline MPC
- Future work: design and optimize secure computation other useful graph analytics